



# Phase transitions and gaps in quantum random energy models

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## HIGHLIGHTS

- Formula for ground and first-excited state energies of Random Energy Models.
- First order quantum phase transition rigorously established.
- Ensemble averaged gap has finite values.
- Non-adiabtic annealing algorithm may have some chances.

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## ABSTRACT

By using a previously established exact characterization of the ground state of random potential systems in the thermodynamic limit, we determine the ground and first excited energy levels of quantum random energy models, discrete and continuous. We rigorously establish the existence of a universal first order quantum phase transition, obeyed by both the ground and the first excited states. The presence of an exponentially vanishing minimal gap at the transition is general but, quite interestingly, the gap averaged over the realizations of the random potential is finite. This fact leaves still open the chance for some effective quantum annealing algorithm, not necessarily based on a quantum adiabatic scheme.

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## 1. Introduction

The perspective to realize a physical device representing a quantum computer (QC) has motivated a fervent research activity concerning the algorithms that could best exploit the intrinsic quantum properties of such a machine as opposed to the classical ones of current computers. In particular, there has been a growing interest toward the possibility to use quantum annealing (QAn) [1–3] as an alternative to simulated thermal annealing [4]. A pictorial viewpoint in fact suggests that in order to get the ground state (GS) of a given classical Hamiltonian  $\mathbf{V}$ , the thermal fluctuations, introduced to avoid the system to be trapped in local minima, could be replaced by quantum fluctuations able to outperform the former due to tunneling effects. Usually, QAn is associated with quantum adiabatic (QAd) algorithms [5–7]. The idea is to implement an interpolating Hamiltonian  $\mathbf{H}(\Gamma) = \mathbf{V} + \Gamma\mathbf{K}$ , where  $\mathbf{K}$  is an operator which does not commute with  $\mathbf{V}$ . The adiabatic theorem ensures that for sufficiently slow changes of the parameter  $\Gamma$  the interpolating system remains in its GS so that the original GS of  $\mathbf{V}$  can be recovered in the limit  $\Gamma \rightarrow 0$ . However, for many interesting problems  $\mathbf{V}$ , the interpolating

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Hamiltonian  $\mathbf{H}(\Gamma)$  is likely to undergo a first order quantum phase transition at some value  $\Gamma = \Gamma_c$ , where the energy gap  $\Delta$  between the first excited state (FES) and the GS becomes exponentially small in the system size  $N$  [8–10]. In this case, a QAd decrease of  $\Gamma$  starting from some value  $\Gamma > \Gamma_c$ , where the GS of  $\mathbf{H}(\Gamma)$  is found with ease, requires an exponentially long time. Otherwise, the system evolves into a combination of several instantaneous eigenstates of  $\mathbf{H}(\Gamma)$  and when  $\Gamma \rightarrow 0$  there is a finite probability to attain a state of  $\mathbf{V}$  different from its GS. For  $\mathbf{V}$  with a glassy energy landscape, “quantum is better” may be untrue [11].

The aim of this paper is threefold, we show that: (i) this phase transition scenario, characterized by a normal (paramagnetic) phase, for  $\Gamma > \Gamma_c$ , and a condensed phase, for  $\Gamma < \Gamma_c$ , is in fact universal, being valid for any hopping operator  $\mathbf{K}$  and any potential operator  $\mathbf{V}$ ; (ii) in the case of random potentials, where one must perform a quenched average over the different realizations of  $\mathbf{V}$ , or, instances, the average of the GS energy determines  $\Gamma_c$  (see Eq. (2)), the average of the gap remains finite and, with respect to  $\Gamma$ , is constant in the condensed phase, and linear in the normal phase; (iii) for any instance there exists a value  $\Gamma_{\min}$  where the gap is exponentially small with the system size  $N$ , and  $\Gamma_{\min} \rightarrow \Gamma_c$  for  $N \rightarrow \infty$ . All the theoretical analysis is supported by unbiased numerical simulations. In the conclusions we shortly discuss the implications of points (i)–(iii) which make impossible to realize an efficient quantum adiabatic algorithm, but still leave open the chance for some effective more complicated quantum annealing scheme [12].

We analyze the case in which  $\mathbf{V}$  is a generic random potential with a discrete or continuous distribution of the levels. For any choice of  $\mathbf{K}$ , provided that  $\mathbf{K}$  has zero diagonal elements in the representation in which  $\mathbf{V}$  is diagonal,  $\mathbf{H}(\Gamma) = \mathbf{V} + \Gamma\mathbf{K}$  is a quantum random energy model (QREM) belonging to the class of systems studied in [13]. The term QREM refers to the quantum counterpart of the classical random energy model (REM), the model defined without the “hopping” operator  $\mathbf{K}$ , namely,  $\mathbf{H} = \mathbf{V}$ . In its original version, the REM was proposed with  $\mathbf{V}$  having Gaussian distributed levels and represents a well known toy model for spin glasses [14]. The corresponding QREM has been first studied perturbatively and numerically in [8].

In Ref. [13] we have exactly characterized the GS and found a sufficient condition for the existence of a first order quantum phase transition for systems in which  $\mathbf{V}$  is an arbitrary random potential. Here, we extend these results to study in detail both the GS and the FES energies of a generic QREM.

## 2. Ground state of random potential systems

In [13] we have determined the exact GS of a class of Hamiltonian models defined by an arbitrary hopping operator  $\mathbf{K}$ , i.e., an off diagonal matrix of dimension  $M$ , and a random potential  $\mathbf{V}$ , i.e., a diagonal matrix with  $M$  i.i.d. random values  $V$  extracted according to an arbitrary probability distribution  $P(V)$ . As usual, we will denote the system corresponding to a particular realization of the  $V$  values as an *instance* and  $\mathbb{E}(\cdot) = \int \cdot P(V) dV$  will stand for the expectation over all possible instances. For the mentioned class of models, in the *thermodynamic limit* the energy  $E_0$  of the GS is related to the lowest level  $E_0^{(0)}$  of the hopping operator by

$$\int \frac{P(V)}{E_0 - V} dV = \frac{1}{E_0^{(0)}}, \quad E_0 \leq \mathbb{E}(V_{(1)}), \quad (1)$$

where  $\mathbb{E}(V_{(k)})$  is the expectation of the  $k$ th order statistic associated with the distribution  $P(V)$ , i.e., the expectation of the  $k$ th smallest value among the  $M$  values of  $V$  drawn according to  $P(V)$  [15]. Note that, whereas  $E_0^{(0)}$  is deterministic,  $E_0$  is stochastic and Eq. (1) is actually an equation for the expectation  $\mathbb{E}(E_0)$ . However, we assume that  $E_0$  is self-averaging and this justifies the above notation.

The derivation of Eq. (1) follows from an exact probabilistic representation of the time evolution of a quantum system in terms of a proper collection of independent Poisson processes [16–18]. In fact, by using this representation, the lowest eigenvalue of  $\mathbf{H} = \mathbf{V} + \Gamma\mathbf{K}$  can be expressed as the solution of a scalar equation written in terms of the asymptotic probability density of the potential and hopping frequencies (frequencies of the values of  $\mathbf{V}$  and  $\mathbf{K}$  realized in the probabilistic representation of an infinitely long time evolution) [13]. It happens that, for the above mentioned class of systems with a random potential, as well as for the uniformly fully connected models, in the thermodynamic limit the asymptotic probability density of the potential and hopping frequencies is exactly given by a multinomial. The equation for the ground-state level of  $\mathbf{H}$  then greatly simplifies and, in the thermodynamic limit, takes on the form of Eq. (1) [13]. We assume a non-trivial thermodynamic limit, namely, a limit in which the lowest levels of  $\mathbf{H}$ ,  $\mathbf{V}$  and  $\mathbf{K}$  diverge with the same speed (the most interesting case being the one in which such levels are all extensive in  $N$ ). Apart from this, the distribution  $P(V)$  remains completely arbitrary.

Eq. (1) has a simple interpretation: the average of the inverse hopping energy, estimated as  $1/(E_0 - V)$ , must coincide with the inverse of the actual hopping energy, namely,  $1/E_0^{(0)}$ , with the constraint that  $E_0$  does not exceed the averaged minimum potential. It may happen that the solution of the integral problem in Eq. (1) becomes incompatible with the constraint  $E_0 \leq \mathbb{E}(V_{(1)})$ . This may occur, for instance, when some parameter of the model is changed below a threshold, e.g.,  $\Gamma < \Gamma_c$  for the Hamiltonian  $\mathbf{H}(\Gamma) = \Gamma\mathbf{K} + \mathbf{V}$ . Then, the GS of an instance condensates, for  $\Gamma < \Gamma_c$ , into the eigenstate of  $\mathbf{V}$  corresponding to the minimum value  $V$  realized in that instance. As a result,  $E_0(\Gamma) = \mathbb{E}(V_{(1)})$  for any  $\Gamma < \Gamma_c$  and we obtain a condensation in the space of states [19]. Note that each instance condensates into a different state, however, this state corresponds, for all instances, to that of minimum potential. A sufficient condition for this first order quantum phase transition to take place is that  $P(\mathbb{E}(V_{(1)})) \rightarrow 0$  in the thermodynamic limit [13].

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