Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 936 (2018) 364–382

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Gravitational properties of the Proca field

Talal Ahmed Chowdhury [a], Rakibur Rahman [a,b,∗], Zulfiqar Ali Sabuj [c]

[a] *Department of Physics, University of Dhaka, Dhaka 1000, Bangladesh*
[b] *Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Am Mühlenberg 1, D-14476 Potsdam-Golm, Germany*
[c] *Department of Theoretical Physics, University of Dhaka, Dhaka 1000, Bangladesh*

Received 28 August 2018; accepted 14 September 2018
Available online 17 September 2018
Editor: Stephan Stieberger

Abstract

We study various properties of a Proca field coupled to gravity through minimal and quadrupole interactions, described by a two-parameter family of Lagrangians. Stückelberg decomposition of the effective theory spells out its model-dependent ultraviolet cutoff, parametrically larger than the Proca mass. We present pp-wave solutions that the model admits, consider linear fluctuations on such backgrounds, and thereby constrain the parameter space of the theory by requiring null-energy condition and the absence of negative time delays in high-energy scattering. We briefly discuss the positivity constraints—derived from unitarity and analyticity of scattering amplitudes—that become ineffective in this regard.

1. Introduction

Fundamental particles relevant to our world must interact with gravity because of its universal nature. For gravitational interactions in flat space, massive fields of arbitrary spin are immune from such severe restrictions as afflict their massless counterparts (see for example [1] for a review). Indeed massive particles do couple to gravity, in particular at the cubic level, giving rise to nontrivial gravitational form factors. To be specific, a massive particle of spin $s \geq 1$ may possess

∗ Corresponding author.
E-mail address: rahmanrakib@gmail.com (R. Rahman).

https://doi.org/10.1016/j.nuclphysb.2018.09.009

as many as $2s$ mass multipole moments.[1] Intuitively, what makes even a fundamental particle behave like an extended object sensitive to tidal forces is its non-zero Compton wavelength, which sets an intrinsic size.

When it comes to massive particles of low spin, such as the Proca field, there seem to be no issues with coupling to gravity. Minimal coupling does not lead to pathologies like Velo–Zwanziger acausality [1]. Neither do non-minimal couplings, consistent with the symmetries of the theory, pose any obvious problems. Of course, the theory will have a cutoff, which—depending on the model—may be parametrically smaller than the Planck scale. Can all such effective field theories be embedded in weakly coupled ultraviolet completions? The answer is expected to be in the negative, from arguments involving unitarity and analyticity of scattering amplitudes [4] or time delays in high-energy scattering [5].

In this article we will consider the gravitational interactions of a massive vector field. The Einstein–Proca theory [6] and its ghost-free generalizations [7–12] have generated a lot of interest in recent years, especially because they provide with an attractive framework for cosmology [13–17] and astrophysics [18–25] (see [26] for a recent review). In this context, one may investigate the Proca self interactions [27,28] or analyze the quantum aspects [29,30]. Sidestepping these interesting directions, we will study the effective field theory of generalized Einstein–Proca actions containing only up to quadratic terms in the Proca field. Explicitly, we will consider the following two-parameter family of Lagrangians:

$$\mathcal{L} = \sqrt{-g}\left[\tfrac{1}{2}M_P^2 R - \tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{1}{2}m^2 A^2 - \tfrac{1}{2}\alpha G_{\mu\nu}A^\mu A^\nu + \tfrac{1}{4}\beta\Lambda^{-2}L_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}\right], \qquad (1)$$

which is an effective field theory of a real Proca field A_μ coupled to Einstein gravity that contains dimensionless parameters α and β, and has an ultraviolet cutoff Λ, where $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ is the Faraday tensor, $G_{\mu\nu}$ the Einstein tensor, and $L_{\mu\nu\rho\sigma}$ the double dual of the Riemann tensor:

$$L^{\mu\nu}{}_{\rho\sigma} = \tfrac{1}{4}\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\rho\sigma\gamma\delta}R_{\alpha\beta}{}^{\gamma\delta} = -R^{\mu\nu}{}_{\rho\sigma} + 4\,\delta^{[\mu}_{[\rho}R^{\nu]}{}_{\sigma]} - R\,\delta^{\mu}_{[\rho}\delta^{\nu}_{\sigma]}. \qquad (2)$$

The particular choice of Eq. (1) corresponds to the most general Lagrangian with non-minimal couplings bilinear in the vector field and linear in curvature, such that higher-order derivative terms are absent in the equations of motion [10,31,32]. Moreover, when expanded around flat space, this Lagrangian will contain all possible Proca-graviton-Proca cubic couplings. Note that there are only two such nontrivial couplings: one with at most two derivatives and another with four derivatives [33,34]. The first one is encoded by minimal coupling, whereas the second by non-minimal quadrupole coupling to the Riemann tensor.[2] Non-minimal couplings to the Ricci tensor and the scalar curvature do appear in our action (1) but they result in trivial cubic interactions in flat space. Yet the inclusion of such terms is well justified. While the $R_{\mu\nu}A^\mu A^\nu$-term is a natural consequence of the ambiguity in minimal coupling prescription since covariant derivatives do not commute, the other terms are essential for having second-order equations of motion in curved space [10], thanks to the transversality properties: $\nabla^\mu G_{\mu\nu} = 0,\ \nabla^\mu L_{\mu\nu\rho\sigma} = 0$.

We would like to study various aspects of the effective field theory described by the generalized Proca model (1). While it is natural to have $\mathcal{O}(1)$ values of α and β, small values of α and m/M_P are technically natural [35] since there is a $U(1)$ symmetry enhancement when these parameters are zero. We will therefore assume that

[1] This count follows from considering the matrix element of the stress-energy tensor between two spin-s states [2]. The multipole expansion of the time-time component in terms of spherical tensors, for example, contains $(2s+1)$ nontrivial pieces, whereas a mass dipole moment is not physically meaningful [3].

[2] Let us recall that a massive spin-1 field may possess only monopole (mass) and quadrupole moments.