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Composite fermions and their pair states in a strongly-coupled Fermi liquid

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Abstract

Our goal is to understand the phenomena arising in optical lattice fermions at low temperature in an external magnetic field. Varying the field, the attraction between any two fermions can be made arbitrarily strong, where composite bosons form via so-called Feshbach resonances. By setting up strong-coupling equations for fermions, we find that in spatial dimension $d > 2$ they couple to bosons which dress up fermions and lead to new massive composite fermions. At low enough temperature, we obtain the critical temperature at which composite bosons undergo the Bose–Einstein condensate (BEC), leading to BEC-dressing massive fermions. These form tightly bound pair states which are new bosonic quasi-particles producing a BEC-type condensate. A quantum critical point is found and the formation of condensates of complex quasi-particles is speculated over.

1. Introduction

The attraction between any two fermions can be tuned, as a function of an external magnetic field, and be made so strong that the coupling constant reaches the unitarity limit of infinite s-wave scattering length "a" via a Feshbach resonance. At that point, a smooth BCS–BEC

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crossover takes place, the Cooper pairs which form in the weak-coupling limit at low temperature and make the system a BCS superconductor, become so strongly bound that they behave like bosonic quasi-particles with a pseudogap at high temperature T^*, and form a new type of BEC at the critical temperature T_c. The recent article [1] reviews the successful progresses of BCS–BEC theories and experiments of dilute Fermi gases, whose thermodynamics can be expressed as scaling functions of a and T, independently of all microscopic details. In this letter, as opposed to dilute Fermi gases, we study strongly interacting fermions in an optical lattice for ongoing experiments [2] and yet completely understood theoretical issues, such as quasi-particle spectra, phase structure and critical phenomena, as well as thermodynamical and transport properties, which can be very different from that of better-studied dilute Fermi gases. We use the approach of strong-coupling expansion to find the massive spectra of not only composite bosons but also composite fermions, and obtain the critical line and phase diagram in the strong-coupling region. Some preliminary discussions are presented on the relevance of our results to experiments.

2. Lattice fermions

We consider fermions in an underlying lattice with a spacing ℓ. In order to address strong-coupling fermions at finite temperature T, we incorporate the relevant s-wave scattering physics via a "ℓ_0-range" contact potential in the Hamiltonian for spinor wave function $\psi_{\uparrow,\downarrow}(i)$, which represents a fermionic neutral atom of fermion number "e" that we call "charge", and $\psi^\dagger_{\uparrow,\downarrow}(i)$ represents its "hole" state "$-e$",

$$\beta\mathcal{H} = \beta \sum_{i,\sigma=\uparrow,\downarrow} (\ell^d)\psi^\dagger_\sigma(i)\Big[-\nabla^2/(2m\ell^2) - \mu\Big]\psi_\sigma(i)$$
$$- g\beta\sum_i(\ell^d)\psi^\dagger_\uparrow(i)\psi^\dagger_\downarrow(i)\psi_\downarrow(i)\psi_\uparrow(i), \tag{1}$$

$\beta = 1/T$, each fermion field $\psi_\sigma(i)$ of length dimension $[\ell^{-d/2}]$, mass m and chemical potential μ is defined at a lattice site "i". The index "i" runs over all lattice sites. The Laplace operator ∇^2 is defined as ($\hbar = 1$)

$$\nabla^2\psi_\sigma(i) \equiv \sum_{\hat\ell}\Big[\psi_\sigma(i+\hat\ell) + \psi_\sigma(i-\hat\ell) - 2\psi_\sigma(i)\Big], \tag{2}$$

where $\hat\ell = 1,\dots,d$ indicate the orientations of lattice spacing to the nearest neighbors. Tuned by optical lattice and magnetic field, the s-wave attraction between the up- and down-spins is characterized by a bare coupling constant $g(\ell_0) > 0$ of length dimension $[\ell^{d-1}]$ and the range $\ell_0 < \ell$.

Inspired by strong-coupling quantum field theories [3,4], we calculate the two-point Green functions of composite boson and fermion fields to effectively diagonalize the Hamiltonian into the bilinear form of these composite fields, and find the composite-particle spectra in the strong-coupling phase.

3. Composite bosons

We first consider a composite bosonic field $\mathcal{C}(i) = \psi_\downarrow(i)\psi_\uparrow(i)$ and study its two-point function on a lattice [4],