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Partial structural restoring of two-qubit transferred state

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We consider the communication line with two-qubit sender and receiver, the later is embedded into the four-qubit extended receiver. Using the optimizing unitary transformation on the extended receiver we restore the structure of the non-diagonal part of an arbitrary initial sender's state at the remote receiver at certain time instant. Obstacles for restoring the diagonal part are discussed. We represent examples of such structural restoring in a communication line of 42 spin-1/2 particles.

1. Introduction

The problem of remote state creation originates from the problem of pure state transfer formulated by Bose [1] and becomes an attracting branch in the area of quantum information processing. Apparently, the state initially created at the sender of a communication line can not be transferred to the receiver unless special protocols are implemented. Among the first protocols we refer to that of perfect state transfer [2–4], remote boundary [5,6] and optimized boundary [7–11] state transfer. Later the remote state creation protocols have been proposed and first realized for the photon systems [12–14], where photons are considered as a basic couriers of quantum information over a long distance. However, the short distance information transfer in quantum information devices can be based on different objects, such as spin chains. As for remote creating a one-qubit state in a spin system, the creatable region in the receiver's state space can be completely described [15,16] because the one-qubit state-space is parametrized with only three parameters. Thus the one-to-one mapping

$$\text{initial sender's state} \;\rightarrow\; \text{creatable receiver's state} \tag{1}$$

is established in that case. The complete characterization of creatable region in higher dimensional state-space is much more complicated. Even two-qubit state depends on 15 parameters, so that the mapping (1) can be hardly visualized. Although we can construct certain families of states in this case, like Werner states [17] in Ref. [18], finding the protocol allowing a more careful control of the link between the initial sender's state and the creatable receiver's state is meaningful.

A method of such control is proposed in Ref. [19,20] where the creation of a two-qubit block-scaled states is considered. In this case the receiver's state defers from the sender's one by the factor ahead of certain blocks of the density matrix. These blocks are multiple-quantum (MQ) coherence matrices. Remember that the n-order coherence matrix collects those elements of the density matrix which are responsible for the state-transitions changing the z-projection of the total spin by n). The feature of these blocks is that they evolve independently provided that the dynamics is described by the Hamiltonian conserving the z-projection of the total spin momentum. However, the protocol proposed in that paper requires a special initial state and is not applicable to an arbitrary one. As a result, each MQ-coherence matrix caries at most one arbitrary parameter.

In this paper we modify the mentioned protocol by implementing the extended receiver (the subsystem at the receiver side embedding the receiver itself [21]) and the fixed optimizing unitary transformation on it. We emphasize that, being fixed, the optimizing unitary

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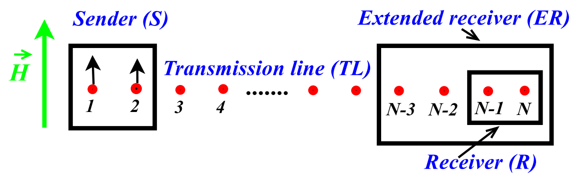

Fig. 1. Communication line including the sender (S), transmission line (TL), receiver (R) and extended receiver (ER). $\vec{H}$ is the external magnetic field.

transformation represents a part of the protocol and remains the same for any transferred state. As a result we manage to structurally reconstruct the non-diagonal part of the initial sender's density matrix in the receiver's density matrix at certain time instant, i.e., the non-diagonal elements of the receiver's density matrix become proportional to the appropriate elements of the sender's initial density matrix in our protocol. We also discuss the obstacle arising in restoring the diagonal elements of the initial sender's density matrix.

Let us mention another aspect of our protocol. Apparently, if there is no optimizing unitary transformation, then the elements of the sender's initial density matrix appear in the receiver's density matrix as linear combinations. Thus, the problem of state transfer reduces to the system of linear algebraic equations for the elements of the sender's density matrix which is solvable in general [22]. However, the quantum-mechanical solution of this system in spin-1/2 communication line is not proposed. Implementing the optimizing unitary transformation, we solve the nondiagonal part of this system and find scaled nondiagonal elements of the sender's initial density matrix. Therefore our protocol contributes into the problem of solving the linear algebraic equations via quantum-mechanical methods [23].

The paper is organized as follows. The proposed model of a communication line is described in Sec. 2. The general protocol of structural restoring of a two-qubit state, including the optimization of the time instant for state registration and construction of the optimal unitary transformation on the four-qubit extended receiver, is proposed in Sec. 3. Examples of numerical structural restoring of non-diagonal elements of a two-qubit initial sender's state in the communication line of $N = 42$ nodes are represented in Sec. 4. General conclusions are given in Sec. 5.

2. Model

We consider the communication line consisting of two-qubit sender S and two-qubit receiver R embedded into the four-node extended receiver ER, which is connected to the sender through the transmission line TL, see Fig. 1. The spin dynamics is governed by the XX-Hamiltonian with the dipole-dipole interaction

$$H = \sum_{j>i} D_{ij}(I_{ix}I_{jx} + I_{iy}I_{jy}), \tag{2}$$

$$[H, I_z] = 0, \tag{3}$$

where $D_{ij} = \frac{\gamma^2\hbar}{r_{ij}^3}$ is the coupling constant between the ith and jth nodes, γ is the gyromagnetic ratio, r_{ij} is the distance between the ith and jth nodes, $I_{i\alpha}$ ($\alpha = x, y, z$) is the projection operator of the ith spin on the α axis and $I_z = \sum_i I_{iz}$. We also consider the tensor-product initial state

$$\rho(0) = \rho^{(S)}(0) \otimes \rho^{(TL)} \otimes \rho^{(ER)}(0), \tag{4}$$

where $\rho^{(S)}(0)$ is an arbitrary initial state of the sender S, $\rho^{(TL)}$ and $\rho^{(ER)}$ are the ground states (states without excitations) of the transmission line and extended receiver respectively:

$$\rho^{(TL)} = \mathrm{diag}(1, 0, \dots), \quad \rho^{(ER)} = \mathrm{diag}(1, 0, \dots). \tag{5}$$

Then

$$\rho^{(R)}(t) = \mathrm{Tr}_{rest}\Big(\tilde{V}(t)\rho(0)\tilde{V}^+(t)\Big), \quad \tilde{V}(t) = e^{-iHt}. \tag{6}$$

Here the trace is over all the nodes of the communication line except the receiver.

In our protocol we use the expansion of the density matrices in the sums of the multiple-quantum (MQ) coherence matrices [24] which read in the two-qubit case as

$$\rho^{(S)} = \sum_{k=-2}^{2} \rho^{(S;k)}, \quad \rho^{(R)} = \sum_{k=-2}^{2} \rho^{(R;k)}, \tag{7}$$

where the MQ coherence matrices $\rho^{(S;k)}$ collect terms responsible for the state transfers changing the z-projection of the total spin momentum by k.

We emphasize that the commutation condition (3) together with the initial condition (5) (where the initial density matrices $\rho^{(TL)}$ and $\rho^{(ER)}$ include only the zero-order coherence matrix) provides transferring the MQ-coherence matrices from the sender to the receiver without mutual interaction [19].