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Topological phase transition in the quasiperiodic disordered Su–Schriffer–Heeger chain

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ABSTRACT

We study the stability of the topological phase in one-dimensional Su–Schrieffer–Heeger chain subject to the quasiperiodic hopping disorder. Two different hopping disorder configurations are investigated, one is the Aubry–André quasiperiodic disorder without mobility edges and the other is the slowly varying quasiperiodic disorder with mobility edges. Interestingly, we find topological phase transitions occur at the critical quasiperiodic disorder strengths which have an exact linear relation with the dimerization strengths for both disorder configurations. We further investigate the localized property of the Su–Schrieffer–Heeger chain with the slowly varying quasiperiodic disorder, and identify that there exist mobility edges in the spectrum when the dimerization strength is unequal to 1. These interesting features of models will shed light on the study of interplay between topological and disordered systems.

1. Introduction

Topological insulators [1–4] (TIs), a unique class of electronic materials, have not only a bulk gap but also symmetry-protected gapless states localized at the sample boundaries. Topological features of TIs are expected to be immune to perturbations of the fluctuation and the environmental noise, and thereby have great potential applications in quantum information processing. Since the effect of disorder is inevitable in real materials, physical properties of topological systems in the presence of disorder have drawn considerable attention in the past decades [5–8]. The combination of topology and disorder can induce rich novel quantum phenomenons. For weak disorder, the robustness of topological states has been demonstrated for various topological systems, especially the quantum spin Hall states [9]. With the increase of the disorder strength, the transition from topological phase to topologically trivial phase can occur. Besides destroying topological states, the medium strength disorder can induce the so called topological Anderson insulator [10–14] (TAI), i.e., a topologically trivial state is driven into a topological state by disorder.

However, the direct observation of the influence of disorder on TIs is difficult due to the difficulty to precisely control the disorder in solid-state experiments. Benefited from the development in the manipulation of ultracold atoms, exploring both disorder and topology via the quantum simulation in artificial systems has become exercisable. In one-dimensional (1D) system the topological/trivial feature of an insulator is completely determined by the presence/absence of the chiral symmetry. The Su–Schrieffer–Heeger (SSH) model [15–18] is one of the most simple and widely studied models belonging to the BDI symmetry class, which hosts two topologically distinguishable phases characterized by the winding number. Using a combination of Bloch oscillations and Ramsey interferometry, Ref. [19] measures the Zak phase to study the topological feature of Bloch bands in a dimerized optical lattice described by 1D SSH model. A very recent work [20], in which a 1D chiral symmetric SSH chain with controllable off-diagonal (hopping) disorder is synthesized based on the laser-driven ultracold atoms [21], systematically explores the disorder effect on topology. They observe the robustness of topological state immune to weak random hopping disorder, and also the topologically non-trivial to trivial transition at very strong hopping disorder. More interestingly, they find the TAI phase in which the band structure of a topologically trivial chain is driven to a non-trivial one by adding the random hopping disorder.

On the other hand, the bichromatic optical lattices [22–24] with incommensurate wavelengths have also attracted enormous attention in the study of Anderson localization. A notable work experimentally investigates the localization property of 1D Aubry-André (AA) model [25] by use of ultracold atoms in the incommensurate/quasiperiodic optical lattice [26]. The AA model has a self-dual symmetry and can undergo a sharp localization transition at the self-dual point without mobility edges. There also exists a class of systems with slowly varying quasiperiodic disorder which can host

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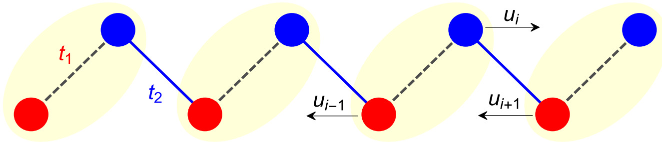

Fig. 1. (Color online.) Schematics of 1D SSH chain with the displacement u_i. The system consists of two sublattices indicated by red and blue filled circles respectively. Hopping amplitudes are staggered by t_1 (red dashed line) and t_2 (blue solid line).

mobility edges [27,28]. Motivated by the highly precise control in these ultracold atomic experiments, an interesting question can be raised: what is the fate of the topological non-trivial state when the quasiperiodic hopping disorder is introduced in 1D SSH chain, besides the random hopping disorder? To examine this question, we propose two quasiperiodic disordered SSH models with two different disorder configurations and investigate how the topological phase transition occurs.

The rest of the paper is organized as follows. In Sec. 2, we investigate the topological phase transition in 1D SSH chain by adding the AA quasiperiodic disorder. In Sec. 3, we investigate the topological phase transition in 1D SSH chain by adding the slowly varying quasiperiodic disorder. We also investigate the localized property (mobility edges) of this system besides the topological phase transition. The conclusion is summarized in Sec. 4.

2. The AA quasiperiodic disorder

The SSH model describes a 1D chain of spinless fermions with alternating hopping strengths between the neighboring tight-binding lattices. For a chain with L lattices and open boundary conditions, the Hamiltonian of the model is expressed as

$$\hat{H} = -\sum_{i}^{L} (t_{i,i+1}\hat{c}_{i+1}^{\dagger}\hat{c}_i + H.c.), \tag{1}$$

where $\hat{c}_i^{\dagger}$ ($\hat{c}_i$) is the fermion creation (annihilation) operator of the i-th lattice, $t_{i,i+1} = t + a(u_{i+1} - u_i)$ is the nearest-neighbor hopping amplitude, a is the displacement coupling constant and u_i is the configuration coordinate for the displacement of the i-th lattice. In the ideal limit, the displacement is of the perfectly periodic form, $u_{i+1} - u_i = (-1)^i \frac{\lambda}{a}$. Thus, the nearest-neighbor hopping amplitude becomes $t_{i,i+1} = t + (-1)^i \lambda$, where the hopping unit t and the dimerization strength λ are both constants. This is the ordinary SSH model. When $\lambda > 0$ the system is in the topological phase with the presence of twofold-degenerate zero-energy edge states at two ends, whereas when $\lambda < 0$ the system is in the topologically trivial phase without the presence of zero-energy edge states.

When the displacement becomes disordered, i.e., $u_{i+1} - u_i = (-1)^i \frac{\lambda+\delta_i}{a}$, Eq. (1) can be written as

$$\hat{H} = -\sum_{i}^{\frac{L}{2}} (t_1\hat{c}_{2i-1}^{\dagger}\hat{c}_{2i} + H.c.) - \sum_{i}^{\frac{L}{2}-1} (t_2\hat{c}_{2i}^{\dagger}\hat{c}_{2i+1} + H.c.), \tag{2}$$

where the intracell hopping $t_1 = t - \lambda - \delta_i$ and the intercell hopping $t_2 = t + \lambda + \delta_i$ with $\delta_i = \delta\cos(2\pi\beta i + \phi)$ represent the AA quasiperiodic disorder, see Fig. 1 for illustration. The total Hamiltonian still maintains the chiral symmetry. In this paper we concentrate on the stability of the topological phase, so we only focus on the situation with $\lambda > 0$ hereafter. A typical choice of the parameters is $\delta > 0$, $\beta = (\sqrt{5} - 1)/2$ and $\phi = 0$. For convenience, $t = 1$ is set as the energy unit.

By numerically diagonalizing the Hamiltonian (2), we can get the eigenvalues (denoted by E) and the wave-functions (denoted by ψ) of the system. We show the spectrum when $\lambda = 0.5$ under the open boundary condition in Fig. 2, where there is a regime with nonzero energy gaps in the range $\delta < 1.5$ and there are zero-energy modes in the midgap. As long as the chiral symmetry is preserved, zero-energy modes cannot be removed by the weak disorder. To show the difference between wave-functions with the zero-energy and nonzero-energy modes clearly, we plot the spatial distributions of wave-functions for the midmost excitations (the 5000th and 5001th eigenvalues) with different δ's. When $\delta = 1.4$, the energy gap is still finite, and the wave-functions with zero-energy modes are located at the left (right) end of the chain and decay very quickly away from the left (right) edge with no overlap, as shown in Fig. 2. However, when $\delta = 1.6$, the energy gap is closed, the zero-energy modes disappear and the amplitudes of the wave-functions of the midmost excitations overlap together and are located within a finite range of the whole chain. Therefore, these results demonstrate that the system can undergo a transition from a topological phase to a topologically trivial phase when the strength of the quasiperiodic disorder δ exceeds a certain level.

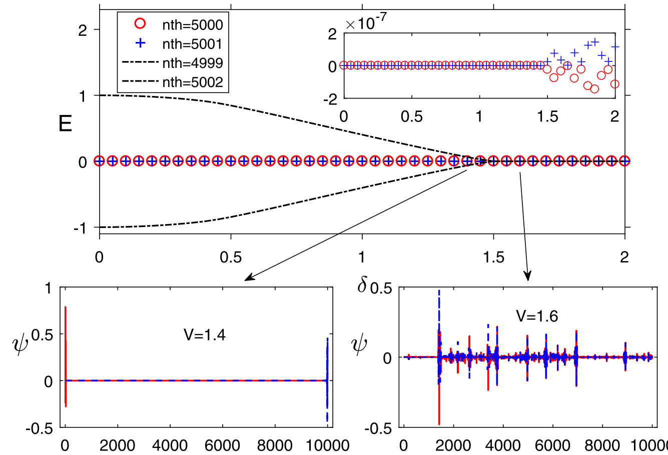

Fig. 2. (Color online.) The spectrum (only the middle four eigenvalues are shown) of the Hamiltonian (2) with $\lambda = 0.5$ as a function of δ under the open boundary condition. The total number of sites is set as $L = 10000$ hereafter. The 5000th and 5001th eigenvalues stay at zero until $\delta = 1.5$, see the blow up of the inset. The spatial distributions of ψ for the 5000th and 5001th eigenvalues with various δ are shown in the lower figures. The lower left picture corresponds to ψ of zero-energy modes in the topological region, and the lower right picture corresponds to ψ of nonzero-energy modes in the topologically trivial region.

We now wonder if there exists a fixed value of δ which denotes the gap-closing point [29]. Due to the chiral symmetry, the eigenvalues appear in pairs. We arrange the eigenvalues in the ascending order and use E' to denote the smallest eigenvalue which is larger than the zero energy. Thus, $\Delta_g = 2E'$ can explicitly determine the gap-closing point and denote the topological phase boundary between different topological phases due to the bulk-boundary correspondence. In Fig. 3, we plot the variation of energy gap Δ_g versus δ for different λ's. It is clearly shown that there are finite gaps as δ is smaller than a critical value, i.e. $\delta < 1 + \lambda$, whereas the energy gaps vanish when $\delta > 1 + \lambda$. To visualize the result better we make a finite size analysis of the scaling behavior of Δ_g for $\lambda = 1$ as a function of the inverse of system size in the inset of Fig. 3. It exhibits an oscillating behavior at $\delta = 1.8$, 1.9, which indicates that the energy gap is finite in the regime with $\delta < 1 + \lambda$. Whereas the energy gap approaches zero at $\delta = 2.0$, 2.1, which indicates that $\delta = 1 + \lambda$ indeed denotes the gap-closing point. To strength the validity of our conclusion, we also systematically calculate the energy gap with other sets of δ and λ, and find the similar behaviors.

In general, the topological phase transition is characterized by the change of the topological invariant. Beyond the translational