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Asymptotically exact codimension-four dynamics and bifurcations in two-dimensional thermosolutal convection at high thermal Rayleigh number: Chaos from a quasi-periodic homoclinic explosion and quasi-periodic intermittency

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Abstract

Using a perturbation method, we solve asymptotically the nonlinear partial differential equations that govern double-diffusive convection (with heat and solute diffusing) in a two-dimensional rectangular domain near a critical point in parameter space where the linearized operator has a quadruple-zero eigenvalue. The asymptotic solution near this codimension-four point is found to depend on two slow-time-dependent amplitudes governed by two nonlinearly-coupled Van der Pol-Duffing equations. Through numerical approximation of the 3-dimensional Poincaré map in the four-dimensional state space of the amplitude equations, we detect and analyze the bifurcations of the amplitude equations as the thermal Rayleigh number, R_T, is increased (for $R_S \ll R_T$, the solute Rayleigh number) with all other parameters fixed. The bifurcations observed include: Hopf, pitchfork and Neimark-Sacker bifurcations of limit cycles, symmetric and asymmetric saddle-node bifurcations of 2-tori, and reverse torus-doubling cascades. In addition, chaotic solutions are found numerically to emerge via two different types of routes: (1) a route involving a homoclinic explosion in the Poincaré map and; (2) type-I intermittency routes near saddle-node bifurcations of 2-tori. The homoclinic explosion occurs when two unstable 2-tori form homoclinic connections with a saddle limit cycle, thereby creating a homoclinic butterfly in the Poincaré map that leads to a discrete Lorenz-like attractor.

Keywords: Double-diffusive convection; Codimension-four unfolding; Nonlinearly-coupled Van der Pol-Duffing equations; Discrete Lorenz-like attractors; Quasi-periodic homoclinic explosion; Quasi-periodic intermittency.

1. Introduction

The partial differential equations that govern two-dimensional, double-diffusive, thermosolutal convection of a fluid in a rectangular box provide a rich stage in which to study nonlinear phenomena. In this convection system, the two scalar fields, temperature and solute, diffuse in the fluid. The imposed fixed temperature and solute gradients on the horizontal sides of the box have a destabilizing and stabilizing effect, respectively, and lead to the competition of stability and instability in the fluid behavior. Exact solutions of the partial differential equations in an appropriate infinite-dimensional function space setting are difficult to generate analytically (due to the nonlinearity of the governing equations). However, near bifurcation points, the infinite-dimensional dynamics can

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