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On interval estimates of perturbations of generalized eigenvalues for diagonalizable pairs ☆

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ABSTRACT

On generalized eigenvalue perturbation bounds for diagonalizable matrix pairs upper bounds are discussed all the time. In this paper we mainly consider not only upper bounds but also lower bounds of perturbation of generalized eigenvalues for diagonalizable matrix pairs. Sharper upper bounds and sharp lower bounds on perturbation of generalized eigenvalues for diagonalizable matrix pairs are obtained in terms of the distances of two points in Grassmann manifold. The main results improve the existing results of Chen (2007) [1] and numerical examples illustrate the efficiency of the theoretical results.

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1. Introduction

Consider an $n \times n$ matrix pair $A - \lambda B$, where A, B are constant matrices. The pair $\{A, B\}$ is said to be regular if $det(A - \lambda B) \neq 0$ for all complex λ. Denote by

$$\mathbf{G} = \{(\alpha, \beta) \neq (0, 0) : \text{ for all complex } \alpha, \beta\}.$$

The pair $(\alpha, \beta) \in \mathbf{G}$ is called a generalized eigenvalue of a regular pair $\{A, B\}$ if $det(\beta A - \alpha B) = 0$. The set of generalized eigenvalues of the regular pair $\{A, B\}$ is denoted by $\lambda(A, B)$. The $n \times n$ regular matrix pairs $A - \lambda B$ and $C - \tilde{\lambda} D$ are diagonalizable, if there exist $n \times n$ invertible matrices S, Q, X, Y such that

$$A = S\Lambda Q, \quad B = S\Omega Q, \quad C = X\Gamma Y, \quad D = X\Delta Y, \tag{1}$$

where

$$\Lambda = diag(\alpha_1, \alpha_1, \ldots, \alpha_n), \quad \Omega = diag(\beta_1, \beta_2, \ldots, \beta_n),$$
$$\Gamma = diag(\gamma_1, \gamma_2, \ldots, \gamma_n), \quad \Delta = diag(\delta_1, \delta_2, \ldots, \delta_n),$$

and $\alpha_i, \beta_i, \gamma_i, \delta_i, 1 \leq i \leq n$ are complex numbers. Especially, when Q and Y are unitary, $\{A, B\}$ and $\{C, D\}$ are normal pairs. Definite matrix pencils, perhaps the most important class of matrix pencils, are always diagonalizable. Generalized eigenvalues of diagonalizable matrix pairs play vital roles in many applications such as control theory [4], model reduction [5], classification and characterization of gene expression data [9], image processing [6], and many more. Perturbation analysis of generalized eigenvalues for diagonalizable matrix pairs has raised many attentions during these decades due to some applications [2,7,11–19,21]. For example, in signal processing a mathematical entity called matrix pair has been utilized by many researchers in array processing and spectral estimation [10]. The diagonalizable matrix pair is simply a linearly combined two matrices, i.e., $A_1 - zA_2$ where z is a scalar variable, and A_1, A_2 are two n by n matrices. In applications, the matrix pair can generally be decomposed into the following form:

$$A_1 - zA_2 = (B_1 + E_1) - z(B_2 + E_2),$$

where E_1 and E_2 are small perturbation matrices due to some kinds of errors and noise. A_1 and A_2 can be two data matrices constructed directly from a data sequence or two covariance matrices with estimated noise covariance matrices removed. E_1 and E_2 represent small residue errors. $\{B_1, B_2\}$ stands for the noiseless matrix pair. The desired values will be called the desired generalized eigenvalues (GEs), which are denoted by $z_1, z_2, \cdots$. The desired GEs contain the desired information like the directions of wave arrivals and the signal poles. Because of the noise or perturbation matrices E_1 and E_2, only noisy estimates of the desired GEs can be obtained from A_1 and A_2. The above

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