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Thermoelectroelastic solutions for surface bone remodeling under axial and transverse loads

Qing-Hua Qin[a,b,*], Chuanyong Qu[a], Jianqiao Ye[c]

[a]*Department of Mechanics, Tianjin University, Tianjin, 300072, PR China*
[b]*Department of Engineering, Australian National University, Canberra, Australia*
[c]*School of Civil Engineering, The University of Leeds, Leeds, LS2 9JT, UK*

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Abstract

Theoretical prediction of surface bone remodeling in the diaphysis of the long bone under various external loads are made within the framework of adaptive elastic theory. These loads include external lateral pressure, electric and thermal loads. Two solutions are presented for analyzing thermoelectroelastic problems of surface bone remodeling. The analytical solution that gives explicit formulation is capable of modeling homogeneous bone materials, while the semi-analytical solution is suitable for analyzing inhomogeneous cases. Numerical results are presented to verify the proposed formulation and to show the effects of mechanical, thermal and electric loads on surface bone remodeling process.

Keywords: Bone surface remodeling; Piezoelectric; Thermal field

1. Introduction

The investigation of remodeling properties and smart behavior of bone tissue and their applications to biomedical engineering by considering its coupled elastic, magnetic, electric behavior has received a considerable interest among the scientists of different fields in the past decades [1–5]. Early in 1950s, Fukada and Yasuda [1, 2] found that some living bone and collagen have piezoelectricity. Later on, Gjelsvik [6] presented a physical description of the remodeling of bone tissue, in terms of very simplified form of the linear theory of piezoelectricity. Williams and Breger [7] explored the applicability of stress gradient theory for explaining the experimental data for a cantilever bone beam subjected to constant end load and showed that the approximate gradient theory is in good agreement with the experimental data. Guzelsu [8] presented a piezoelectric model for analyzing cantilever dry bone beam subjected to a vertical end load. Johnson et al. [9] further addressed the problem of dry bone beam by presenting some theoretical expressions for piezoelectric response to cantilever bending of the beam. Demiray [10] gave some theoretical descriptions on electro-mechanical remodeling models of bones. Aschero et al. [11] investigated converse piezoelectric effect of fresh bone by using a high-sensitive dilatometer. They provided further investigations on piezoelectric properties of bone and presented a set of repeated measurements of coefficient d_{23} on 25 cow bone samples [12]. Fotiadis et al. [13] studied the wave propagation in a long cortical piezoelectric bone with arbitrary cross-section. El-Naggar et al. [14] and Ahmed et al. [15] further obtained an analytical solution on wave propagation in long cylindrical bones with and without cavity. Silva et al. [16] explored physico-chemical, dielectric and piezoelectric properties of anionic collagen

*Corresponding author. Department of Engineering, Australian National University, Canberra, Act 0200, Australia.
Tel.: +61 2 61258274; fax: +61 2 61250506.
E-mail address: qinghua.qin@anu.edu.au (Q.-H. Qin).

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and collagen–hydroxyapatite composites. Recently, Qin and Ye [4] presented a thermoelectroelastic solution for internal bone remodeling. Tsubota and Adachi [17] studied spatial and temporal regulation process of cancellous bone using computer simulation. Most developments in this area can also be found in [3,18]. It can be seen from the discussion above that bone remodeling is a highly organized process, but the mechanisms which determine where, when, and how remodeling occurs are still open questions.

In this work, two solutions for thermoelectroelastic problems of surface bone remodeling, based on the theory of adaptive elasticity [19], are presented to study the effects of mechanical, thermal and electric loads on surface bone remodeling process. The analytical solution is used for investigating surface bone remodeling process on the basis of assuming a homogeneous bone material [4], while the semi-analytical solution is developed for analyzing bone materials that are assumed radially inhomogeneous. Numerical results are presented to show applicability of the proposed solutions and the effect of thermal and electric loads on the bone surface remodeling process.

2. Solution of surface modeling for a homogeneous hollow circular cylindrical bone

2.1. Equation for surface bone remodeling

The equations of the theory of adaptive elasticity of Cowin and vanBuskirk [19] are used and extended to include piezoelectric effect in this study. The remodeling rate equation in cylindrical coordinates is

$$\begin{aligned} U &= C_{ij}(\mathbf{n}, Q)\left\lfloor s_{ij}(Q) - s^0_{ij}(Q)\right\rfloor + C_i\left\lfloor E_i(Q) - E^0_i(Q)\right\rfloor \\ &= C_{rr}s_{rr} + C_{\theta\theta}s_{\theta\theta} + C_{zz}s_{zz} + C_{rz}s_{rz} + C_rE_r \\ &\quad + C_zE_z - C_0, \end{aligned} \tag{1}$$

where $C_0 = C_{rr}s^0_{rr} + C_{zz}s^0_{zz} + C_{\theta\theta}s^0_{\theta\theta} + C_{rz}s^0_{rz} + C_rE^0_r + C_zE^0_z$; U denotes the velocity of the remodeling normal to the surface; C_{ij} and C_i are surface remodeling coefficients.

2.2. Analytical solution

We now consider a hollow circular cylinder of bone, subjected to an external temperature change T_0, a quasi-static axial load P, an external pressure p and an electric potential load φ_a(or/andφ_b). The boundary conditions are

$$\begin{aligned} &T = 0, \quad \sigma_{rr} = \sigma_{r\theta} = \sigma_{rz} = 0, \quad \varphi = \varphi_a \quad \text{at} \quad r = a, \\ &T = T_0, \quad \sigma_{rr} = -p, \quad \sigma_{r\theta} = \sigma_{rz} = 0, \quad \varphi = \varphi_b \\ &\quad \text{at} \quad r = b \end{aligned} \tag{2}$$

and

$$\int_S \sigma_{zz}\mathrm{d}S = -P, \tag{3}$$

where a and b denote, respectively, the inner and outer radii of the bone, and S is the cross-sectional area. For a long bone, it is assumed that except the axial displacement u_z, all displacements, temperature and electrical potential is independent of the z coordinate and that u_z may have linear dependence on z. The solution of displacements u_r, u_z and electric potential φ to the problem above has been discussed elsewhere [4]. For the reader's convenience we list them in Appendix A at the end of this paper.

The strains and electric field intensity can be found by introducing Eqs. (A.7)–(A.10) into (A.2) [the solution given in appendix]. They are, respectively,

$$\begin{aligned} s_{rr} = \frac{1}{F^*_3}&\left(c_{33}\beta^*_1[\beta^*_2T_0 + p(t)] + \varpi\frac{c_{33}c_{12}}{c_{11}}\right. \\ &\left.+\frac{F^*_2T_0 + P(t)}{\pi(b^2 - a^2)}c_{13} - F^*_1T_0c_{13}\right) \\ &-\frac{a^2\beta^*_1[\beta^*_2T_0 + p(t)]}{r^2(c_{11} - c_{12})} + \frac{\varpi\ \ln(r/a)}{c_{11}}, \end{aligned} \tag{4}$$

$$\begin{aligned} s_{\theta\theta} = \frac{1}{F^*_3}&\left(c_{33}\beta^*_1[\beta^*_2T_0 + p(t)] + \varpi\frac{c_{33}c_{12}}{c_{11}}\right. \\ &\left.+\frac{F^*_2T_0 + P(t)}{\pi(b^2 - a^2)}c_{13} - F^*_1T_0c_{13}\right) \\ &+\frac{a^2\beta^*_1[\beta^*_2T_0 + p(t)]}{r^2(c_{11} - c_{12})} + \frac{\varpi[\ln(r/a) - 1]}{c_{11}}, \end{aligned} \tag{5}$$

$$\begin{aligned} s_{zz} = \frac{1}{F^*_3}&\left(\left[F^*_1T_0 - \frac{F^*_2T_0 + P(t)}{\pi(b^2 - a^2)}\right](c_{11} + c_{12})\right. \\ &\left.-2c_{13}\beta^*_1[\beta^*_2T_0 + p(t)] - \frac{2c_{13}c_{12}\varpi}{c_{11}}\right), \end{aligned} \tag{6}$$

$$s_{rz} = -\frac{e_{15}(\varphi_b - \varphi_a)}{rc_{44}\ \ln(b/a)}, \tag{7}$$

$$E_r = -\frac{(\varphi_b - \varphi_a)}{r\ \ln(b/a)}. \tag{8}$$

Substituting (4)–(8) into (1) yields

$$\begin{aligned} U_\mathrm{e} &= N^\mathrm{e}_1\frac{b^2}{b^2 - a^2} + N^\mathrm{e}_2\frac{1}{\ln(b/a)} + N^\mathrm{e}_3\frac{1}{b^2 - a^2} \\ &\quad + N^\mathrm{e}_4\frac{1}{a\ \ln(b/a)} - C^\mathrm{e}_0, \\ U_\mathrm{p} &= N^\mathrm{p}_1\frac{b^2}{b^2 - a^2} + N'^\mathrm{p}_1\frac{a^2}{b^2 - a^2} + N^\mathrm{p}_2\frac{1}{\ln(b/a)} \\ &\quad + N^\mathrm{p}_3\frac{1}{b^2 - a^2} + N^\mathrm{p}_4\frac{1}{b\ \ln(b/a)} + N'_3 - C^\mathrm{p}_0, \end{aligned} \tag{9}$$