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Improved linear programming methods for checking avoiding sure loss

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ABSTRACT

We review the simplex method and two interior-point methods (the affine scaling and the primal-dual) for solving linear programming problems for checking avoiding sure loss, and propose novel improvements. We exploit the structure of these problems to reduce their size. We also present an extra stopping criterion, and direct ways to calculate feasible starting points in almost all cases. For benchmarking, we present algorithms for generating random sets of desirable gambles that either avoid or do not avoid sure loss. We test our improvements on these linear programming methods by measuring the computational time on these generated sets. We assess the relative performance of the three methods as a function of the number of desirable gambles and the number of outcomes. Overall, the affine scaling and primal-dual methods benefit from the improvements, and they both outperform the simplex method in most scenarios. We conclude that the simplex method is not a good choice for checking avoiding sure loss. If problems are small, then there is no tangible difference in performance between all methods. For large problems, our improved primal-dual method performs at least three times faster than any of the other methods.

1. Introduction

In statistical modelling, we often face issues such as limited structural information about dependencies, lack of data, limited expert opinion, or even contradicting information from different experts. Various authors [16,17,14,11] have argued that these issues can be handled by modelling our beliefs using *sets of desirable gambles*. A gamble represents a reward (e.g. monetary) that depends on an uncertain outcome. We can model our beliefs about this outcome by stating a collection of gambles that we are willing to accept. Such set is called a set of desirable gambles. Through duality, every set of desirable gambles is mathematically equivalent to a set of probability distributions.

If there are no combinations of desirable gambles that result in a certain loss, then we say that our set of desirable gambles *avoids sure loss* [16,17]. To verify whether a set of desirable gambles avoids sure loss, we can solve a linear programming problem [14, p. 151].

Linear programs for checking avoiding sure loss have been studied for instance in [15,9]. However, these studies focus on forming linear programs and do not mention which algorithms we should use. In the early '90s, Walley [14, p. 551] mentioned that Karmarkar's method can be considered for solving large linear programs. However, nowadays Karmarkar's method is considered obsolete in favour of other interior point methods such as affine scaling and primal-dual methods [1].

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The simplex method is one of the oldest and simplest methods, and the affine scaling method is an improved version of Karmarkar's method, whilst the primal-dual method is currently considered one of the best general purpose methods. In previous work, we presented an initial comparative study of these three methods for checking avoiding sure loss [8]. In that study, we slightly reduced the size of the problems and proposed two improvements: an extra stopping criterion to detect unboundedness more quickly, and a direct way to calculate feasible starting points. There, we also quantified the impact of these improvements [8, Fig. 1], for the primal-dual method.

In this paper, our main goal is to elaborate on the improvements in [8], and to further develop efficient algorithms for checking avoiding sure loss. In particular, we study also the dual of each linear program, and we generalise the process to find feasible starting points. We also discuss in detail the advantages and disadvantages of each method for checking avoiding sure loss. For benchmarking, we provide a variety of algorithms for generating sets of desirable gambles. In a simulation study, we generate random sets of desirable gambles and assess the impact of our improvements. In addition, we provide proofs for all relevant results, including some that were stated without proof in [8].

The paper is organised as follows. Section 2 gives a brief outline of avoiding sure loss and coherence. Section 3 studies several linear programming problems for checking avoiding sure loss, and we slightly reduce the size of these linear programming problems. Section 4 reviews the simplex, the affine scaling and the primal-dual algorithms, and studies how we can improve these algorithms to check avoiding sure loss. Sections 5 and 6 present some algorithms for generating random sets of desirable gambles. Section 7 compares the efficiency of our improved methods. Section 8 concludes the paper.

2. Desirable gambles and lower previsions

In this section, we explain desirable gambles, lower previsions, avoiding sure loss, coherence, and natural extension [14]. We also introduce the notation used throughout.

2.1. Avoiding sure loss

Let Ω be a finite set of uncertain outcomes. A *gamble* is a bounded real-valued function on Ω. Let $\mathcal{L}(\Omega)$ denote the set of all gambles on Ω. Let $\mathcal{D}$ be a finite set of gambles that a subject decides to accept; we call $\mathcal{D}$ the subject's *set of desirable gambles*. The desirability axioms essentially state that a non-negative combination of desirable gambles should not produce a sure loss [14, §2.3.3]. In that case, we say that $\mathcal{D}$ avoids sure loss.

Definition 1. [14, §3.7.1] A set $\mathcal{D} \subseteq \mathcal{L}(\Omega)$ is said to *avoid sure loss* if for all $n \in \mathbb{N}$, all $\lambda_1, \dots, \lambda_n \geq 0$, and all $f_1, \dots, f_n \in \mathcal{D}$,

$$\max_{\omega\in\Omega}\left(\sum_{i=1}^{n}\lambda_i f_i(\omega)\right) \geq 0. \tag{1}$$

We can also model uncertainty via acceptable buying (or selling) prices for gambles. A *lower prevision* $\underline{P}$ is a real-valued function defined on some subset of $\mathcal{L}(\Omega)$. We denote the domain of $\underline{P}$ by $\operatorname{dom}\underline{P}$. Given a gamble $f \in \operatorname{dom}\underline{P}$, we interpret $\underline{P}(f)$ as a subject's supremum buying price for f.

Definition 2. [14, §2.4.2] A lower prevision $\underline{P}$ is said to *avoid sure loss* if for all $n \in \mathbb{N}$, all $\lambda_1, \dots, \lambda_n \geq 0$, and all $f_1, \dots, f_n \in \operatorname{dom}\underline{P}$,

$$\max_{\omega\in\Omega}\left(\sum_{i=1}^{n}\lambda_i\left[f_i(\omega) - \underline{P}(f_i)\right]\right) \geq 0. \tag{2}$$

Any lower prevision $\underline{P}$ induces a *conjugate upper prevision* $\overline{P}$ on $-\operatorname{dom}\underline{P} := \{-f\colon f \in \operatorname{dom}\underline{P}\}$, defined by $\overline{P}(f) := -\underline{P}(-f)$ for all $f \in -\operatorname{dom}\underline{P}$ [14, §2.3.5]. $\overline{P}(f)$ represents a subject's infimum selling price for f. $\underline{P}$ is said to be *self-conjugate* when $\operatorname{dom}\underline{P} = -\operatorname{dom}\underline{P}$ and $\underline{P}(f) = \overline{P}(f)$ for all $f \in \operatorname{dom} P$. We simply call a self-conjugate lower prevision $\underline{P}$ a *prevision* and write it as P [11, p. 41].

2.2. Coherence

Coherence is another rationality condition for lower previsions and is stronger than avoiding sure loss. Coherence requires that the subject's supremum buying prices for gambles cannot be increased by considering any finite non-negative linear combination of other desirable gambles [14, §2.5.2]. In Section 5, we will use coherent lower previsions to generate sets of desirable gambles that avoids sure loss.