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Recurrence statistics for anomalous diffusion regime change detection

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Abstract

For many real-time series, specific behaviors are observed where the character of the time series changes over time. This temporal evolution may indicate that some properties of the data evolve or fluctuate. One can find such problems in many different applications including physical and biological experiments as well as in technical diagnostics. From the mathematical point of view, this complexity can be considered as a segmentation problem, i.e. extraction of the homogeneous parts from the original data. Most segmentation methods assume that a simple characteristic of the time series changes, for example the mean or the variance. However, many physical applicationsinvolve a more complex situation dealing with transient statistics. Here, a new technique of the critical change point detection is introduced for the case when the data consist of anomalous diffusion processes with transient anomalous diffusion exponents. The precise mathematical formulation of a new statistics based on recurrence statistics is provided. The proposed recurrence analysis counts the number of data points falling into the appropriate circle built from consecutive observations. This approach proves to be helpful in recognizing subdiffusive and superdiffusive regions, which characterize anomalous diffusion behaviors. The main characteristics of the recurrence statistics are presented and the application to the segmentation problem is described. The effectiveness of the proposed technique is validated for a family of classical anomalous diffusive models, namely fractional Brownian motion. Finally, the methodology is applied to biological data exhibiting anomalous diffusion behavior with transient anomalous diffusion exponents.

Keywords: segmentation, anomalous diffusion process, anomalous diffusion exponent, fractional Brownian motion

1. Introduction

For many real data, specific behaviors that indicate the process under consideration is not homogeneous are observed. More precisely, the statistical properties of the data can change over time. In many cases, the observed time series can be modeled by a single model, albeit with a set of parameters that switches between different states. In this type of intermittent processes, an important step in the data analysis is the segmentation, i.e. the extraction of homogeneous parts with the same statistical properties. In general, the criterion for segmentation of real trajectories depends on the specific application. In some cases, the difference between states is visually recognizable, but often advanced processing is required to find the structure transition points. Thus, the key issue is to find a description of the different states that highlights their differences.

In recent years, substantial works on segmentation methods for different applications appeared in the literature. A few interesting applications include condition monitoring [23, 42, 60], biomedical signals (e.g., electrocardiogram) [6, 10, 13, 22, 57, 71, 78], turbulent plasmas [28], speech analysis [35, 45, 50], econometrics [31, 32, 73], and seismic signals [21, 27, 40, 62, 69]. This problem appears also in the motion of individual molecules as observed by single-particle tracking in living cells [29, 56, 54, 37, 59, 38]. Single-particle tracking measurements probe the stochastic motion of molecules as they interact with specific binding partners [63, 81, 72, 36, 83], move between different

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