ARTICLE IN PRESS

Future Generation Computer Systems ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Future Generation Computer Systems

journal homepage: www.elsevier.com/locate/fgcs

Sparsity estimation matching pursuit algorithm based on restricted isometry property for signal reconstruction

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HIGHLIGHTS

- A sparsity estimation method based on the Restricted Isometry Property (RIP) criterion is proposed for determining the number of selection atoms.
- An adaptive adjustment method for step size is proposed to remove mismatching atoms and accelerate the reconstruction process.
- The proposed RSEMP algorithm can provide a high reconstruction quality and a less computational time under either a random measurement matrix or a determinacy measurement matrix when the sparsity is pre-estimated.

ARTICLE INFO

Article history:
Received 10 March 2017
Received in revised form 1 June 2017
Accepted 14 September 2017
Available online xxxx

Keywords:
Signal reconstruction
Matching pursuit
Sparsity estimation
Restricted isometry property criterion

ABSTRACT

Achievement of good reconstruction performance by most of existing greedy algorithms is possible only when signal sparsity has been known well in advance. However, it is difficult in practice to ensure signal sparsity making the reconstruction performance of the greedy algorithms stable. Moreover, some greedy algorithms with previous unknown signal sparsity are time-consuming in the process of adaptive adjustment of signal sparsity, and thereby making the reconstruction time too long. To address these concerns, the greedy algorithm from signal sparsity estimation proposed in this paper. Based on the restricted isometry property criterion, signal sparsity is estimated before atoms selection and the step size of atoms selection adjusted adaptively based on the relations between of the signal residuals in each iteration. The research which solves the problem of sparsity estimation in the greedy algorithm provides the compressed sensing available to the applications where the signal sparsity is un-known. It has important academic and practical values. Experimental results demonstrate the superiority of the performance of proposed algorithm to the greedy algorithms with previous unknown signal sparsity, no matter on the performance stability and reconstruction precision.

1. Introduction

Compressed sensing (CS) is an exciting and rapidly developing research field, arousing wide interests from researchers in the fields of signal processing, statistics, computer science and even the entire scientific community. In recent years, many scholars have carried out many research works on CS theory, which has been applied to various aspects [1–5]. Obviously, the key to the popularity of CS theory lies in the ability of the signal to reconstruct with a high precision after compression and sampling. Especially, in the field of wireless real-time transmission, fast and accurate signal reconstruction algorithm is particularly in urgent demand, and the research on reconstruction algorithm is essential. So reconstruction algorithms become the emphasis in CS theory and attract extensive attentions worldwide [6–9].

CS theory presents the concept that if the signal $y \in R^{m\times 1}$ has at the most K non-zero elements, signal y can be projected by the measurement matrix $\Psi \in R^{s\times m}$ $(s \ll m)$, and then a measurement vector $f \in R^{s\times 1}$ whose dimension is much smaller than y is obtained [10]. That can be expressed as follows:

$$f = \Psi y \tag{1}$$

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https://doi.org/10.1016/j.future.2017.09.034
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In fact, the process of signal projection with the measurement matrix is the process of signal compression. Notably, formula (1) is the underdetermined equations and has innumerability solutions. But y is K-sparse signal, and when the measurement matrix satisfies certain conditions, y can be reconstructed accurately by measurement vector f by solving the minimum value of l_0- norm. The objective function is as follows:

$$\arg\min_{y} \|y\|_0 \; s.t. \; f = \Psi y \tag{2}$$

If the $y \in R^{m\times 1}$ is not a sparse signal, a dictionary base $\Phi \in R^{m\times n}$ is needed and y can be sparsely represented as follow:

$$y = \Phi x \tag{3}$$

According to the CS theory, the mathematical model of CS can be represented as:

$$f = \Psi y = \Psi\Phi x = \Theta x \tag{4}$$

where $\Psi \in R^{s\times m}$ is measurement matrix, Θ is sensing matrix and $\Theta = \Psi\Phi$. It seems that the signal sampling and compression can be performed simultaneously and the signal transmission is no longer following the traditional Nyquist sampling theorem as a result of the emergence of the sensing matrix. Then, the signal y can be reconstructed by solving the following objective function.

$$\arg\min_{x} \|x\|_0 \; s.t. \; f = \Theta x \tag{5}$$

By solving the above l_0-norm minimization problem, the approximate solution $\hat{x}$ of objective function is obtained, and then the signal is reconstructed according to $\hat{y} = \Phi\hat{x}$.

We can conclude that signal reconstruction process based on CS is the process of the recovery of the m-dimension original signal $y \in R^{m\times 1}$ from the s-dimension measurement signal, where $s \ll m$. If y is sparse, the signal reconstruction problem can be translated into solving the l_0- norm minimization problem in formula (2); otherwise, if y is not sparse, the signal reconstruction problem can be translated into solving the l_0- norm minimization problem in formula (5). In other words, no matter whether y is sparse, the signal reconstruction will be translated into solving the l_0- norm minimization problem after compressed sensing. However, solving the minimum value of l_0- norm is a NP-hard problem [11], and accordingly, many scholars have proposed the convex optimization algorithms, combination algorithms and greedy algorithms for solving the suboptimal solution of the objective function (2) or (6) to replace global optimal solution [12–15].

The convex optimization algorithms can translate the l_0-norm minimization problem to the l_1-norm minimization problem, and translate the nonconvex problem into a convex problem, which has been proved by Elad and Bruckstein in 2002 [16], as well as the Donoho D L [17] and Sharon Y [18]. The existing convex optimization algorithms applied to CS include Inner Point algorithm [19], Projected Gradient Methods algorithm [8] and Iterative Thresholding algorithm [20]. Combination algorithms use group testing for reconstructing the highly structured samples of the orthogonal signal. HHS Pursuit [21] and Sub-linear Fourier Transform [22] are the representative ones in the numerous combination algorithms. Compared with convex optimization algorithms, the computing speed of combination algorithms are faster and usually achieve the advantages of sublinear. Greedy algorithms gradually approximate sparse coefficient and the original signal by iteratively pursuing the best matching atomic, and its performance is theoretically close to the result of l_0 norm minimization. The typical greedy algorithms include Orthogonal Matching Pursuit (OMP) algorithm [7], Regularized Orthogonal Matching Pursuit (ROMP) algorithm [23], Compression Sample Matching Pursuit (CoSaMP) algorithm [24], Subspace Pursuit (SP) algorithm [25], Stagewise Orthogonal Matching Pursuit (StOMP) algorithm [26], StagewiseWeak Orthogonal Matching Pursuit (SWOMP) algorithm [9] and Sparsity Adaptive Matching Pursuit (SAMP) algorithm [27].

The conclusion from a further study of the above three kinds of algorithms, is that the reconstruction accuracy of a convex optimization algorithm is higher and the number of measurements is less than a greedy algorithm. However, the considerable reconstruction time of a convex optimization algorithm makes it difficult to apply to the large-scale signal processing applications. The computational complexity of a combination algorithm is lower while the number of measurements is much more than a greedy algorithm, which limits the ability of data compression. A greedy algorithm has significant advantages on implementation and reconstruction time. However, whose global convergence cannot be guaranteed. Generally, by adding certain constraint condition, there is plenty of room for performance improvement on the reconstruction accuracy and reconstruction time of the greedy algorithm. Consequently, this paper mainly focused on the research of the greedy algorithm.

The focus of greedy algorithms is on the choice of the suitable atoms. In other words, how to make rules of atom selection is the hotspot on the greedy algorithms researches. The existing representative greedy algorithms, OMP algorithm, CoSaMP algorithm, SP algorithm and ROMP algorithm need to know signal sparsity in advance to ensure a better reconstruction performance. Unfortunately, in real applications, it is impossible to determine the sparsity of every signals to satisfy the condition of high accuracy reconstruction of greedy algorithms. Hence, some scholars have proposed some greedy algorithms for matching pursuit the termination conditions independent from the signal sparsity, including StOMP algorithm, SWOMP algorithm and SAMP algorithm. StOMP algorithm uses the current residual for determining the threshold of atom selection and pursue multiple atoms in every iteration on the basis of the concept of stagewise orthogonal; however, pursuit of multiple atoms brings in some constraints on the measurement matrix. Afterwards, SWOMP algorithm is proposed for changing the threshold of atom selection and lower the demands on designing the measurement matrix. StOMP algorithm and SWOMP algorithm focus only on the threshold of atom selection without considering the signal sparsity for signal reconstruction. Then, SAMP algorithm is proposed which updates the selection number of atoms based on the current residual in each iteration, and searches the best matching set of atoms iteratively to achieve the adaptive sparsity. However, the process of matching pursuit of SAMP has no theoretical basis, and then its performance is not stable. Therefore, this paper proposes a greedy algorithm named RSEMP algorithm, which takes the Restricted Isometry Property (RIP) criterion as theoretical basis of signal sparsity estimation, using the relations between of the residuals to adjust the step size of the atom selection for pursuing the best matching atoms set to achieve a more excellent reconstruction performance.

2. Signal sparsity estimation based on restricted isometry property

Signal reconstruction with a high precision should follow RIP criterion, which gives the restricted condition between the sensing matrix and the sparse signal. The restricted condition is described as below.

A signal x is said to be K-sparse if x has at most K nonzero coordinates. The sensing matrix Θ is said to satisfy the Restricted Isometry Property of order K with constant $\delta \in (0, 1)$, for any K-sparse vector x ($\|x\|_0 \le K$) [28], we have

$$(1-\delta)\,\|x\|_2^2 \le \|\Theta x\|_2^2 \le (1+\delta)\,\|x\|_2^2 \tag{6}$$

When the original signal is sparse, the dictionary base Φ in the sensing matrix $\Theta = \Psi\Phi$ is an identity matrix, and the sensing