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A no-equilibrium memristive system with four-wing hyperchaotic attractor

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ABSTRACT

A new memristive system is proposed in this paper which can have no equilibrium and a line of equilibrium based on the value of its controlling parameter. Also, changing that parameter can cause the system having both chaotic and hyperchaotic solutions. This system has a multi-wing strange attractor. Dynamical properties of this system such as Lyapunov exponents and bifurcation diagram are calculated. This system belongs to the category of systems with hidden and multistable attractors. A system with all the above-mentioned properties is not common in the literature. Finally, an adaptive sliding mode control method is applied to synchronize this chaotic system.

1. Introduction

Memristor (concatenation of MEMory ResISTOR), is the fourth fundamental circuit element, described by a nonlinear voltage-current relation [1]. Memristive circuits and systems have brought great achievements in simulating processes which needs memory such as biological systems [2] (e.g. learning and associative memory) and designing logical gates. Memristive systems show more complex dynamical behaviors, like chaos, hyperchaos and hidden attractors [3,4], than other common nonlinear systems. Because of applications of these complex behaviors, for example in the image processing [5,6,7], control engineering [8,9,10], electronic engineering [11,12,13] and neural networks [8], designing memristive systems with particular dynamical features attract excessive interest nowadays [14].

Multi-stability is a very important phenomenon which can be observed in some dynamical systems [15,16]. The maximum possible multi-stability is extreme multi-stability which has been observed in chaotic and memristive systems [17,18,19,20,21,22].

Hyperchaotic systems have more complex solutions than chaotic systems. They have more than one positive Lyapunov exponent (LE) i.e. for a four-dimension hyperchaotic system there are two positive LEs, one zero, and one negative LE [23,24,25]. Especial memristive systems and circuits with no equilibrium [26,27,28,29], a line of equilibria [30], different number of wings [11,31,32,33], hidden attractors [34,35,36,37], multistability [38,39,40,41], and extreme multistability [19] have been reported in literature.

Synchronization of chaotic systems has applications in secure communication and cryptography [42,43]. The highly sensitive nature of chaotic systems to initial conditions makes it difficult to synchronize the systems with uncertainties and disturbance. Some well-known ways to synchronize chaotic systems are using active control method [44], sliding mode control [45], Adaptive sliding mode [46,47], etc.

The rest of this paper is organized as follows. In Section 2, the proposed system is introduced. In Section 3, the dynamical properties of the system which includes Lyapunov exponents diagram (Section 3.1, bifurcation diagram (Section 3.2, and multistability analysis (Section 3.3 is presented. Synchronization of this system is divided into the problem statement (Section 4.1 and HMS synchronization (Section 4.2 sections. Also, an electronic circuit of the proposed system is presented in Section 5. Finally, concluding remarks are discussed in Section 6.

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2. Hyperchaotic memristive system (HMS)

A memristive system with a line equilibrium was proposed in [30] which surprisingly can show a four-wing hyperchaotic attractor. A 3D system is considered and introduced with memristor in the second state and the flux that passes through the memristor became a new state variable denoted as w. Also state y is the state voltage v going into the memristor, and let k be a positive parameter indicating the strength of the memristor. In this paper, we propose a modified version of the system proposed in [30] by considering a control parameter. This modified system shows both the line equilibrium and no equilibrium conditions for different values of the control parameter. The hyperchaotic memristive system with the new control parameter is defined as,

$$\begin{aligned} \dot{x} &= ax + byz \\ \dot{y} &= cy + dxz - kyW(w) - TSC \\ \dot{z} &= ez + fxy + gxw \\ \dot{w} &= -y \end{aligned} \quad (1)$$

where $W(w)$ is called the memductance and given by $W(w) = m + 3nw^2$. In Fig. 1, the parameters of the HMS are set to$a = 0.35$, $b = -10$, $c = -0.6$, $d = 0.3$, $e = -1.6$, $f = 2$, $g = 0.1$, $m = 0.1$, $n = 0.01$, $k = 0.2$, $TSC = 0.01$ and initial conditions are considered as $[0.1, 0.1, 0.1, 0.1]$.

The HMS exhibits hyperchaotic attractors without equilibrium points when the control parameter $TSC \neq 0$. When $TSC = 0$, the system has a line equilibrium in $(0, 0, 0, w)$. In [30], it has been shown that this line equilibrium is consisting of infinite unstable saddle points.

3. Dynamical analysis of the HMS

3.1. Lyapunov exponents

We will discuss the dynamics of the HMS for $TSC \neq 0$ when the HMS has no defined equilibrium and hence shows hidden oscillations. The finite time Lyapunov exponents are calculated using the Wolfs algorithm [48] as $L_1 = 0.1032$, $L_2 = 0.0149$, $L_3 = 0$, and $L_4 = -1.996$. Two positive Lyapunov exponents confirm that the HMS shows hyperchaotic behavior when $a = 0.35$, $b = -10$, $c = -0.6$, $d = 0.3$, $e = -1.6$, $f = 2$, $m = 0.1$, $n = 0.01$, $k = 0.2$ and initial conditions are $[0.1, 0.1, 0.1, 0.1]$. It should be noted that there are some important issues about calculating Lyapunov exponents described in [49,50,51,52]. Also note that LEs, computed on a transient chaotic set, may be positive for a very long time, while, finally, the trajectory may converge to stationary point and limit value of LE will be negative. In our calculations, the Lyapunov exponents have been calculated for a duration of 50,000 sec, which seems to be long enough.

3.2. Bifurcation diagram

To investigate the dynamical behavior of the HMS with parameters, we discuss the bifurcation of the system as the parameter increases. In this case our parameter of interest is TSC which controls the system's equilibrium points. The range of the parameter for the bifurcation is taken as $[-0.06, 0.06]$ and the local maxima of the state variable x is plotted as shown in Fig. 2a. The HMS takes a period doubling route to chaos and shows chaotic oscillations for $-0.0508 < TSC < 0.0504$. To show the type of attractor, we investigate the Lyapunov exponents' spectrum in Fig. 2b. It can be seen that the HMS shows two positive LEs in the range $-0.0515 < TSC < 0.05$and thus exhibits a hyperchaotic attractor.

3.3. Multistability analysis

To study the multistability of the HMS, we use the forward (increasing the parameter from minimum to maximum with reinitializing the initial conditions to the end values of state trajectories and plotting the local maxima of the state variables) and backward (decreasing the parameter in discussion from maximum to minimum with reinitializing the initial conditions to the end values of state trajectories and plotting the local maxima of the state variables) bifurcation. Fig. 3a shows the forward (blue dots) and backward (red dots) bifurcation of the HMS while parameter TSC varied between $[-0.06, 0.06]$ and the other parameters are set to $a = 0.35$, $b = -10$, $c = -0.6$, $d = 0.3$, $e = -1.6$, $f = 2$, $m = 0.1$, $n = 0.01$, $k = 0.2$ and $(x(0), y(0), z(0), w(0)) = [0.1, 0.1, 0.1, 0.1]$. We could see a small window $-0.05055 < TSC < 0.0513$ where coexisting attractor with a limit cycle is seen and coexisting attractors are seen for $TSC = 0.0505$ as shown in Fig. 4a. Similarly coexisting period-8 oscillations are seen for $TSC = -0.0508$ as shown in Fig. 4b.

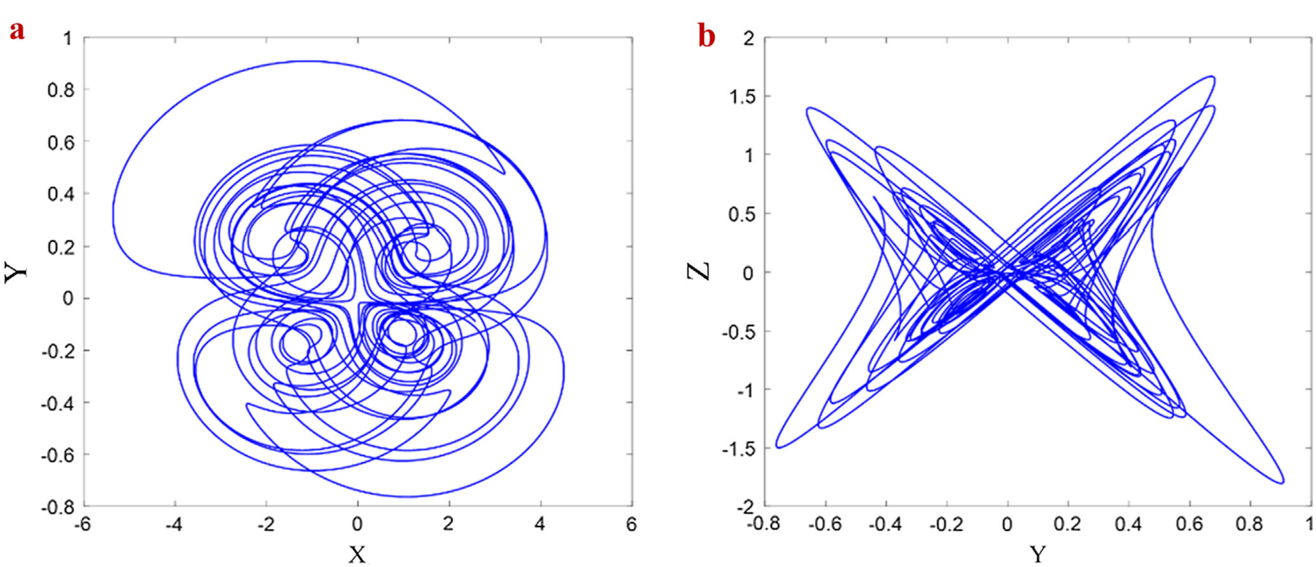

Fig. 1. 2D phase portraits of the HMS shown in the (a) X–Y plane and (b) Y–Z plane when$a = 0.35$, $b = -10$, $c = -0.6$, $d = 0.3$, $e = -1.6$, $f = 2$, $g = 0.1$, $m = 0.1$, $n = 0.01$, $k = 0.2$, $TSC = 0.01$ and $[x(0), y(0), z(0), w(0)] = [0.1, 0.1, 0.1, 0.1]$.