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Integrated approach for computing aggregation weights in cross-efficiency evaluation

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ABSTRACT

Cross-efficiency evaluation is an extension of data envelopment analysis that has been widely used in many different applications aimed at producing a ranking of the set of decision making units. Besides the traditional self-appraisal of units, cross-evaluation methods also take into account peer-appraisals, which are then summarized into an overall performance measure. The standard approach for this aggregation process relies on an equally-weighted average that disregards that some cross-efficiency scores might be considered more relevant or reliable than others. This paper focuses on the aggregation process of cross-efficiency scores and proposes a new approach for deriving meaningful aggregation weights for a more comprehensive evaluation of the units. Our method integrates two complementary perspectives that weights should reflect: the discriminatory ability of the information contained in the cross-efficiency matrix and the relative importance that can be attributed to each of the peer-appraisals. In this sense, the approach presented here provides a more accurate evaluation of the units than previous approaches and therefore it is likely to produce more meaningful rankings. Some numerical examples are provided that validate the approach proposed and examine the results obtained in comparison with previous known methods.

1. Introduction

Data envelopment analysis (DEA), first introduced in [1], is a linear programming technique useful for assessing the relative efficiency of a homogeneous set of decision making units (DMUs) that operate in a production system where multiple inputs are consumed to produce multiple outputs. After four decades of development, there is still intense research activity in the field both at a theoretical as well as empirical level [2], which has proved DEA to be a valuable tool for performance evaluation in many different contexts, with interesting applications in health care, education, banking, manufacturing, etc.

In the traditional DEA model DMUs' performance is assessed using an efficiency score defined as a ratio of a weighted sum of outputs to a weighted sum of inputs. These efficiency scores are obtained through a self-evaluation process, where each DMU is allowed to choose its own set of optimal input and output weights that guarantee a maximum efficiency ratio, as long as the scores of all DMUs calculated from the same weights do not exceed one. According to this evaluation framework, DMUs obtaining a unitary efficiency value are regarded as efficient units whereas DMUs that are unable to attain the maximum efficiency level are considered to perform inefficiently.

This self-assessment scheme with total flexibility in weights selection is especially suitable for the identification of inefficient DMUs, but very commonly too many units are classified as efficient performers not allowing further discrimination or ranking among them, which may result unsatisfactory in several decision contexts aiming at finding the best performer. This lack of discrimination in DEA applications is well documented, particularly when the number of inputs and outputs is too high relative to the number of DMUs being evaluated, and some empirical rules have been suggested to avoid too many units being classified as efficient [3]. Further research has been undertaken with the aim of increasing the discriminative power of DEA. As a result, the traditional model has been extended in different directions (see for example [4,5] and references therein), including for example weight-restriction models, super-efficiency models or common-weight models, although perhaps the methods based on a cross-evaluation approach stand among the most commonly used for ranking DMUs.

Cross-efficiency evaluation, firstly proposed in [6], complements the traditional self-evaluation mode of DEA with a peer-evaluation mode, in such a way that each DMU is also assessed using the most favorable weight set of its peer DMUs. Consequently, in a cross-evaluation framework DMUs act both as evaluated and evaluating units.

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This evaluation scheme, where DMUs are repeatedly assessed using a range of input and output weights instead of a single set of weights, provides a more detailed view of the performance of DMUs, allowing a glimpse of how sensitive the performance assessments of units are to the weight pattern used. Eventually, the scores that a DMU obtains when it is rated by its peers and the self-rated efficiency score are averaged into an overall performance measure that summarizes the different appraisals received by that DMU. As Doyle and Green [7] pose it, cross-evaluation approaches enjoy the same connotations of a democratic process, in the sense that each and every DMU's preferences regarding weights are taking into account within the evaluation procedure, and therefore the results obtained are likely to be considered as a consensual assessment.

Besides this interesting feature, cross-evaluation approaches are found to achieve complete discrimination among DEA-efficient units, which is particularly effective for purposes of ranking. These advantages explain the extensive use of cross-efficiency based approaches in applications involving performance evaluation of DMUs for decision-making within a wide range of fields (see for example, [8–13]). The relevance of this line of research has also been confirmed in the study carried out by Liu et al. [2] who identified cross-efficiency related studies to be one of the more active DEA research subareas in recent years.

Despite the interesting advantages and vast applicability of cross-efficiency evaluation, it has received some critics due to non-uniqueness of cross-efficiency scores. Given that the optimal set of DEA weights selected by each DMU is not necessarily unique, multiple cross-efficiency scores can be obtained depending on the specific optimal solution that the LP solver generates. To overcome this problem, Sexton et al. [6] suggested the use of some secondary objectives to guide the selection of a particular set of optimal weights in a specified direction. Particularly, they proposed the so-called benevolent and aggressive formulations, aimed at finding a set of weights that guarantee the optimal efficiency score of the evaluated DMU while making the others DMUs' cross-efficiencies as large (for benevolent) or small (for aggressive) as possible. Throughout the years, a great deal of research has focused on this topic and many different secondary goal models have been proposed to handle the non-uniqueness issue, either following a benevolent or aggressive strategy [7,14,15] or introducing neutral objectives that avoid taking a position for or against the peers [16,17]. All in all, the introduction of a secondary goal can be seen as an opportunity to intentionally specify a particular strategy for selecting weights that makes the procedure better fit the desired aim of the analysis. Moreover, it has significantly enriched the theoretical development of the methodology.

Less attention has been paid in the literature to the issue of the aggregation of the self and peer evaluated cross-efficiencies into a single cross-efficiency score for each unit. In the standard approach the cross-efficiency score of each DMU is defined as the average of the cross-appraisals received, and although other aggregation measures could also be adopted, they are rarely applied [18]. Particularly, the use of a simple average implicitly assumes that the assessments provided by all the DMUs are equally relevant or reliable. However, this is not necessarily always the case and several arguments can be used to justify that attaching equal aggregation weights to all the cross-efficiency scores may not be completely satisfactory and can fail to reflect the real performance of the evaluated units [19]. In this sense, grounded in the belief that using a weighted average aggregation introduces a higher degree of modeling flexibility that may lead to more accurately assessments of DMUs' performance, some authors have studied the calculation of relative importance weights to be used during the aggregation of cross-efficiency scores, and a few approaches have subsequently been developed that differ in the way the notion of DMUs importance is tackled.

In this work an alternative method is proposed to derive aggregation weights that estimate the relative importance of the cross-efficiencies provided by different DMUs. The rationale of our approach develops from known strategies for weight elicitation in multicriteria decision making, given that the scores that need to be aggregated in a cross-evaluation scheme bear a notable resemblance with the ratings of an alternative set across a number of criteria. Particularly, we suggest considering two components in the definition of the importance weights: an intrinsic component, which is intended to reflect how much information can be inferred from the DMUs appraisals for a discrimination purpose, and a contextual component, which is intended to reflect how relevant or valuable the DMUs appraisals can be considered within the background conditions where the evaluation process takes place.

The rest of the paper is organized as follows. Section 2 describes the traditional cross-evaluation approach and how the aggregation of cross-efficiency scores is tackled in the literature. In Section 3 an alternative approach that computes relative importance weights for cross-efficiency aggregation is presented. Then, some numerical examples from the DEA literature are examined in Section 4 to illustrate the application of our approach in comparison with the traditional average cross-efficiency and other weighted cross-efficiency approaches and finally, some concluding remarks are provided in Section 5.

2. Average and weighted cross-efficiency evaluation

Let us consider n production units or DMUs, each of them being evaluated in terms of r inputs and s outputs. Using the standard notation, let x_{ij} and y_{kj} be nonnegative values denoting respectively the amount of input i consumed and the amount of output k produced by the jth DMU ($i = 1, \dots, r,\ k = 1, \dots, s,\ j = 1, \dots, n$). In this setting, the original DEA ratio model developed in [1] allows each DMU to choose an optimal set of input and output weights to achieve the maximum efficiency score, defined as the ratio of the weighted sum of outputs to the weighted sum of inputs, constrained to no other DMU scoring more than one. Therefore, for each DMU q under evaluation the following non-linear program is formulated:

$$\begin{aligned} E_{qq} = \max \quad & \frac{\sum_{k=1}^{s} u_{kq} y_{kq}}{\sum_{i=1}^{r} v_{iq} x_{iq}} \\ \text{s. t.} \quad & \\ & \frac{\sum_{k=1}^{s} u_{kq} y_{kj}}{\sum_{i=1}^{r} v_{iq} x_{ij}} \le 1 \quad j = 1, \dots n \\ & u_{kq},\ v_{iq} \ge 0 \end{aligned} \tag{1}$$

which can be suitably transformed into an equivalent linear program that is usually known as CCR model, named after its authors Charnes, Cooper and Rhodes:

$$\begin{aligned} E_{qq} = \max \quad & \sum_{k=1}^{s} u_{kq} y_{kq} \\ \text{s. t.} \quad & \sum_{i=1}^{r} v_{iq} x_{iq} = 1 \\ & \sum_{k=1}^{s} u_{kq} y_{kj} - \sum_{i=1}^{r} v_{iq} x_{ij} \le 0 \quad j = 1, \dots n \\ & u_{kq},\ v_{iq} \ge 0 \end{aligned} \tag{2}$$

Problem (2) must be solved n times in order to obtain a set of optimal weights $u_{kq}^*,\ v_{iq}^*$ ($i = 1, \dots, r,\ k = 1, \dots, s$) and the efficiency scores E_{qq} for all the units analyzed $q \in \{1, \dots, n\}$, allowing a classification of the DMU set into efficient ($E_{qq} = 1$) and non-efficient ($E_{qq} < 1$) units. When the most preferred weights for a given DMU are used to compute an efficiency score for the other DMUs we obtain the so-called cross-efficiency values, $E_{jq} = \frac{\sum_{k=1}^{s} u_{kq}^* y_{kj}}{\sum_{i=1}^{r} v_{iq}^* x_{ij}}$ ($j = 1, \dots, n$) which represent the evaluation of DMU j under the perspective of DMU q. The resulting $n \times n$ values can be gathered into a matrix that will be referred to as cross-efficiency matrix (CEM), which is represented in Table 1. As seen,