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Definite integrals for aggregating continuous interval-valued intuitionistic fuzzy information

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ABSTRACT

Atanassov's interval-valued intuitionistic fuzzy set (A-IVIFS), as an extension of Atanassov's intuitionistic fuzzy set (A-IFS), uses the membership function and the non-membership function to generate interval values to describe things' characteristics. To simplify the expression of interval-valued intuitionistic fuzzy number (IVIFN), the concept of simplified interval-valued intuitionistic fuzzy number (SIVIFN) is proposed. Recently, some experts have proposed a series of ways to fuse Atanassov's interval-valued intuitionistic fuzzy information. However, we find that those techniques can't aggregate those IVIFNs that spread all over a region. So in this paper, we try to find a new technique to solve the above-mentioned issue. Firstly, based on SIVIFNs, we propose the interval-valued intuitionistic fuzzy definite integral, analyze its properties and construct the relation between the proposed definite integral of SIVIFNs and that of IFVs proposed by previous researches, and then, based on the definite integral, we construct an interval-valued intuitionistic fuzzy definite integral (IIFDI) operator. Next, a technique is proposed to aggregate amounts of SIVIFNs by statistical methods. At last, an example about satisfaction evaluation of patient on medical system in China is given to show the effectiveness and practicability of our technique.

1. Introduction

Fuzzy set theory [30], proposed by Zadeh in 1965, uses fuzzy set to describe fuzzy information and has solved a series of decision making problems [17,18]. However, the membership function in fuzzy set can only generate a single-valued membership degree, but can't express other objection and hesitance information. So Atanassov defined intuitionistic fuzzy set (A-IFS) [1], which is expressed by a membership degree, a non-membership degree and an indeterminancy degree (it is noted that the word “intuitionistic” is “Atanassov's intuitionistic” [7] throughout the paper, but for brevity, we omit “Atanassov” before the name “intuitionistic” hereafter). To some extent, the A-IFS can describe things' vagueness more specifically and comprehensively. To make A-IFS become visualizable, Atanassov [4] gave the geometrical interpretation of the elements of A-IFSs and the operations over them, which has generated a lot of the new ideas in intuitionistic fuzzy sets theory. Moreover, A-IFS has been universally applied into various fields, such as decision making [6,9,12,13,22,31], clustering analysis [19,20,23,32,33] and so on. Among these references, Liu et al. [12,13] proposed Heronian aggregation operators and intuitionistic fuzzy interaction partitioned Bonferroni mean operators under the intuitionistic fuzzy environment to solve the problems of decision making, respectively. And Ban [5] introduced fuzzy integrals on A-IFSs which enriched intuitionistic fuzzy measures to some extent.

With the continuous development of intuitionistic fuzzy set theory, the experts gradually find that the membership degree and the non-membership degree are difficult to be expressed by exact real numbers in some occasions. Thus, Atanassov and Gargov [2] gave a definition of interval-valued intuitionistic fuzzy set (A-IVIFS), in which the membership function and the non-membership function can generate interval values to describe things' characteristics. Then for convenience, Xu [24] simplified IVIFS by an ordered pair, called interval-valued intuitionistic fuzzy number (IVIFN) and introduced some basic operational laws. To some extent, the introduction of IVIFN provides new ideas for expressing ambiguous information and solving the decision making problems. Due to the advantages of IVIFSs, some experts have tried to fuse interval-valued intuitionistic fuzzy (IVIF) information by different kinds of operators to make decisions

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[8,11,14,21,24–26,34]. For example, Xu developed a series of aggregation operators for IVIFNs, such as the interval-valued intuitionistic fuzzy weighted averaging (IIFWA) operator [24], the interval-valued intuitionistic fuzzy hybrid averaging (IIFHA) operator [25], etc. Liu [14] proposed two interval-valued intuitionistic fuzzy operators based on the power average operator and Heronian mean operator, and then use them to aggregate interval-valued intuitionistic fuzzy information.

By referring to a large number of relevant documents, it is not difficult to find that most of the aggregation operators only fuse the discrete IVIF information and manipulate a limited number of data, but can not deal with a great number of data. This shortcoming of the traditional aggregation operators makes against aggregating those IVIFNs that spread all over a region. So in the following, to overcome the above-mentioned issue, we mainly concentrate on looking for an operator and finding a technique to aggregate continual IVIF information. In mathematics, the definite integral in real number field is a tool that can integrate all continual data in a closed region. Motivated by this idea, we shall extend the definite integral to the IVIF environment, and based on which we construct an interval-valued intuitionistic fuzzy definite integral (IIFDI) operator. Then by exploration, we find that there are some relations between the definite integral of SIVIFNs and that of IFVs in Ref. [15]. For the discrete data, a SIIFWA operator can be expressed by two IFWA operators, for the continuous data, a definite integral operator of SIVIFNs can also be expressed by that of IFNs. The proposed operator can more efficiently aggregate the continuous IVIF information and solve the above-mentioned problem. For convenience, the simplified interval-valued intuitionistic fuzzy number (SIVIFN) [16] is the smallest unit of calculation.

The remainder of this paper is arranged as follows: Section 2 introduces some basic knowledge of IFVs and SIVIFNs and their operational laws. Section 3 gives the definition of the interval-valued intuitionistic fuzzy definite integral. In Section 4, we analyze its properties and construct the relation between the definite integral of SIVIFNs and that of IFVs. In Section 5, by restricting the weight density function, we construct an interval-valued intuitionistic fuzzy definite integral (IIFDI) operator based on the definite integral. Section 6 gives an example to illustrate the validity and practicability of the proposed operator. The paper ends up with some conclusions in Section 7.

2. Preliminaries

Let X be a fixed set. In Ref. [1], an IFS is defined on X as $A = \left\{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\right\}$, where $\mu_A(x)$ and $\nu_A(x)$ respectively indicate the membership degree and non-membership degree of the element x to A and satisfy the condition: $\mu_A(x) \in [0,1]$, $\nu_A(x) \in [0,1]$, and $0 \le \mu_A(x)+\nu_A(x) \le 1$, for all $x \in X$. For convenience, Xu [27] defined the intuitionistic fuzzy value (IFV) as $\alpha = (\mu_\alpha, \nu_\alpha)$, where $\mu_\alpha \in [0,1]$, $\nu_\alpha \in [0,1]$, and $\mu_\alpha+\nu_\alpha \le 1$, which is the base element of IFS. And then the concept of intuitionistic fuzzy pair [3] defined by Atanassov et al. was utilized as the evaluation of some object or process. Later based on the operation rules of IFSs defined in [1,10], some operations of IFVs were proposed by Xu and Yager in Refs. [27,28] as follows:

Definition 2.1. [27,28]
Let $\alpha = (\mu_\alpha, \nu_\alpha)$, $\alpha_1 = \left(\mu_{\alpha_1}, \nu_{\alpha_1}\right)$ and $\alpha_2 = \left(\mu_{\alpha_2}, \nu_{\alpha_2}\right)$ be three IFVs. Then some relevant operations have the following form:

(1) $\alpha_1 \cap \alpha_2 = \left(\min\left\{\mu_{\alpha_1}, \mu_{\alpha_2}\right\}, \max\left\{\nu_{\alpha_1}, \nu_{\alpha_2}\right\}\right)$;
(2) $\alpha_1 \cup \alpha_2 = \left(\max\left\{\mu_{\alpha_1}, \mu_{\alpha_2}\right\}, \min\left\{\nu_{\alpha_1}, \nu_{\alpha_2}\right\}\right)$;
(3) $\alpha_1 \oplus \alpha_2 = \left(\mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1}\mu_{\alpha_2}, \nu_{\alpha_1}\nu_{\alpha_2}\right)$;
(4) $\lambda\alpha = \left(1-(1-\mu_\alpha)^\lambda, \nu_\alpha\right), \lambda > 0$.

Definition 2.2. [27]
Let $\alpha = (\mu_\alpha, \nu_\alpha)$, $\alpha_1 = \left(\mu_{\alpha_1}, \nu_{\alpha_1}\right)$ and $\alpha_2 = \left(\mu_{\alpha_2}, \nu_{\alpha_2}\right)$ be three IFVs. The score function and the accuracy function are denoted by $s(\alpha) = \mu_\alpha - \nu_\alpha$ and $h(\alpha) = \mu_\alpha + \nu_\alpha$, respectively. $s(\alpha_1)$ and $s(\alpha_2)$ are the scores of α_1 and α_2, $h(\alpha_1)$ and $h(\alpha_2)$ are the accuracy degrees of α_1 and α_2. Then

(1) If $s(\alpha_1) < s(\alpha_2)$, the IFV α_1 is smaller than α_2, denoted by $\alpha_1 < \alpha_2$;
(2) If $s(\alpha_1) = s(\alpha_2)$, then
  1) If $h(\alpha_1) = h(\alpha_2)$, the IFV α_1 is equal to α_2, denoted by $\alpha_1 = \alpha_2$;
  2) If $h(\alpha_1) > h(\alpha_2)$, the IFV α_1 is larger than α_2, denoted by $\alpha_1 > \alpha_2$.

Theorem 2.1. [15]

Let $\alpha = (\mu_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \nu_\beta)$ be two IFVs, then

(1) If $\lambda_1 > \lambda_2$, then $\lambda_1\alpha > \lambda_2\alpha$, $\lambda_1, \lambda_2 \ge 0$;
(2) If $\mu_\alpha \ge \mu_\beta$, $\nu_\alpha \le \nu_\beta$, then $\lambda\alpha > \lambda\beta$, $\lambda \ge 0$.

To express vague information flexibly, Atanassove and Gargov [2] proposed the concept of interval-valued intuitionistic fuzzy set (A-IVIFS), denoted by $\mathbf{A} = \left\{(x, \mu_{\mathbf{A}}(x), \nu_{\mathbf{A}}(x)) | x \in X\right\}$, where $\mu_{\mathbf{A}}(x) \subset [0,1]$, $\nu_{\mathbf{A}}(x) \subset [0,1]$ and $\sup\mu_{\mathbf{A}}(x) + \sup\nu_{\mathbf{A}}(x) \le 1, x \in X$. Later, Ren et al. [16] simplified the IVIFS as $\mathbf{A} = \left\{< x, \alpha, \beta > | x \in X\right\}$, where α and β are two IFVs, with the condition $\alpha \cup \beta = \alpha$ or $\alpha \cap \beta = \beta$. They called $\boldsymbol{a} = \left[\alpha, \beta\right]$ a simplified interval-valued intuitionistic fuzzy number (SIVIFN). Since the SIVIFN is simple and easy to express IVIF information, in this paper, we construct the definite integral based on SIVIFNs, and all the letters representing IVIFN or SIVIFN are bold.

Definition 2.3. [16]
Let $\boldsymbol{a} = \left[\alpha, \beta\right]$, $\boldsymbol{a}_1 = \left[\alpha_1, \beta_1\right]$, and $\boldsymbol{a}_2 = \left[\alpha_2, \beta_2\right]$ be three SIVIFNs, then some basic operations among them are defined as the following forms: