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Correct energy evolution of stabilized formulations: The relation between VMS, SUPG and GLS via dynamic orthogonal small-scales and isogeometric analysis. II: The incompressible Navier–Stokes equations

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Highlights

- The energy behavior of the RBVMS for LES of incompressible flow is examined.
- We present a novel GLS method with dynamic small-scales that has correct energy behavior.
- The method conserves mass and linear and angular momentum and has divergence-free small-scales.
- The correct-energy demand creates a link between VMS, GLS and SUPG.
- The method shows excellent accuracy on a 3D Taylor–Green vortex flow case.

Abstract

This paper presents the construction of a correct-energy stabilized finite element method for the incompressible Navier–Stokes equations. The framework of the methodology and the correct-energy concept have been developed in the convective–diffusive context in the preceding paper [M.F.P. ten Eikelder, I. Akkerman, Correct energy evolution of stabilized formulations: The relation between VMS, SUPG and GLS via dynamic orthogonal small-scales and isogeometric analysis. I: The convective–diffusive context, Comput. Methods Appl. Mech. Engrg. 331 (2018) 259–280]. The current work extends ideas of the preceding paper to build a stabilized method within the variational multiscale (VMS) setting which displays correct-energy behavior. Similar to the convection–diffusion case, a key ingredient is the proper dynamic and orthogonal behavior of the small-scales. This is demanded for correct energy behavior and links the VMS framework to the streamline-upwind Petrov–Galerkin (SUPG) and the Galerkin/least-squares method (GLS).

The presented method is a Galerkin/least-squares formulation with dynamic divergence-free small-scales (GLSDD). It is locally mass-conservative for both the large- and small-scales separately. In addition, it locally conserves linear and angular momentum.

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The computations require and employ NURBS-based isogeometric analysis for the spatial discretization. The resulting formulation numerically shows improved energy behavior for turbulent flows comparing with the original VMS method.

Keywords: Stabilized methods; Energy decay; Residual-based variational multiscale method; Orthogonal small-scales; Incompressible flow; Isogeometric analysis

1. Introduction

The creation of artificial energy in numerical methods is undesirable from both a physical and a numerical stability point of view. Therefore methods precluding this deficiency are often sought after. This work continues the construction of the correct-energy displaying stabilized finite element methods. The first episode [1] exposes the developed methodology in the convective–diffusive context. The current study deals with the incompressible Navier–Stokes equations and is the second piece of work within the framework. The setup of this paper is closely related to that of [1]. In particular, the *correct-energy* demand is the same, thus it represents that the method (i) does not create artificial energy and (ii) closely resembles the energy evolution of the continuous setting. The precise definition is stated in Section 4. What sets the Navier–Stokes problem apart from convection–diffusion case is the inclusion of the incompressibility constraint. In this work we use a divergence conforming basis which allows exact pointwise satisfaction of this constraint. This is considered a beneficial property. Therefore it is added as a design criterion. In a two-phase context this property is essential for correct energy behavior [2].

1.1. Contributions of this work

This paper derives a novel VMS formulation which exhibits the correct energy behavior and to this purpose combines several ingredients. The final formulation is summarized in Appendix A. The new method is a residual-based approach that employs (i) dynamic behavior of the small-scales, (ii) solenoidal NURBS basis functions and (iii) a Lagrange-multiplier construction to ensure the incompressibility of the small-scale velocities. The formulation is of skew-symmetric type, rather than conservative, which is motivated by both the correct-energy demand and its improved behavior in the single scale setting (i.e. the Galerkin method) [3]. Moreover, the formulation reduces to a Galerkin formulation in case of a vanishing Reynolds number due to a Stokes-projector. The use of dynamic small-scales, firstly proposed in [4], is also driven from an energy point of view. In addition, it leads to global momentum conservation and the numerical results of [5] show improved behavior of the dynamic small-scales with respect to their static counterpart.

1.2. Context

This work falls within the variational multiscale framework [6,7]. The basic idea of this method is to split solution into the large/resolved-scales and small/unresolved-scales. The small-scales are modeled in terms of (the residual of) the large scales and substituted into the equation for the large-scales. This approach was first applied in a residual-based LES context to incompressible turbulence computations in [8]. The VMS methodology has enjoyed a lot of progress since then. For an overview of the development consult the review paper [9].

Our work is not the first to analyze the energy behavior of the VMS method. A spectral analysis of the VMS method can be found in [10]. That paper proves dissipation of the model terms under restrictive conditions. Additional to the optimality projector, they require L_2-orthogonality of the large- and small-scales. This condition naturally leads to the use of spectral methods.

Principe et al. [11] provide a precise definition of the numerical dissipation within the variational multiscale context for incompressible flows. Equally important, they numerically show that the concept of dynamic small-scales, which we apply in this work, is able to model turbulence.

Colomés et al. [12] assess the performance of several VMS methods for turbulent flow problems and provide an energy analysis of these methods. They conclude that algebraic subgrid scales (ASGS) and orthogonal subscales (OSS) yield similar results, whereas the latter one is more convenient in terms of numerical performance.

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