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A hybrid adaptive low-Mach number/compressible method: Euler equations

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ABSTRACT

Flows in which the primary features of interest do not rely on high-frequency acoustic effects, but in which long-wavelength acoustics play a nontrivial role, present a computational challenge. Integrating the entire domain with low-Mach-number methods would remove all acoustic wave propagation, while integrating the entire domain with the fully compressible equations can in some cases be prohibitively expensive due to the CFL time step constraint. For example, simulation of thermoacoustic instabilities might require fine resolution of the fluid/chemistry interaction but not require fine resolution of acoustic effects, yet one does not want to neglect the long-wavelength wave propagation and its interaction with the larger domain.

The present paper introduces a new multi-level hybrid algorithm to address these types of phenomena. In this new approach, the fully compressible Euler equations are solved on the entire domain, potentially with local refinement, while their low-Mach-number counterparts are solved on subregions of the domain with higher spatial resolution. The finest of the compressible levels communicates inhomogeneous divergence constraints to the coarsest of the low-Mach-number levels, allowing the low-Mach-number levels to retain the long-wavelength acoustics. The performance of the hybrid method is shown for a series of test cases, including results from a simulation of the aeroacoustic propagation generated from a Kelvin–Helmholtz instability in low-Mach-number mixing layers. It is demonstrated that compared to a purely compressible approach, the hybrid method allows time-steps two orders of magnitude larger at the finest level, leading to an overall reduction of the computational time by a factor of 8.

1. Introduction

Many interesting fluid phenomena occur in a regime in which the fluid velocity is much less than the speed of sound. Indeed, it is possible to make a distinction between scales of fluctuations, depending on how a hydrodynamic fluid element is sensitive to acoustic perturbations. Acoustic waves that do not carry enough energy to perturb a flow are referred to short-wavelengths. In contrary, long-wavelengths refer to large scale motions where acoustic and hydrodynamic fluctuations can interact. Low-Mach-number [1–3] schemes exploit the separation of scales between acoustic and advective motions; these methods calculate the convective flow field but do not allow explicit propagation of acoustic waves. Their compu-

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tational efficiency relative to explicit compressible schemes results from the fact that the time step depends on the fluid velocity rather than sound speed. However, there is a class of problems for which the small-scale motions can be adequately captured with a low-Mach-number approach, but which require in addition the representation of long wavelength acoustic waves. This paper introduces a computational methodology for accurately and efficiently calculating these flows.

An important example of this type of flow is thermoacoustic instabilities in large scale gas turbine engines. In these engines the region where the burning takes place can be modeled using a low-Mach-number approach, since the short-wavelength acoustic waves generated by the heat release do not carry sufficient information or energy to be of interest. Low-Mach-number modeling of turbulent combustion has been demonstrated to be an efficient way to generate accurate solutions [4–9]. However, in large burners, under certain conditions the long-wavelength acoustic waves that emanate from the burning region can reflect from the walls of the burner and impinge on the burning region, generating thermoacoustic instabilities which can be violent enough to disrupt the flame, as well as lead to mechanical failures or excessive acoustic noise [10–15]. There is currently a great deal of interest in the problem of how to control the instabilities through passive or active control mechanisms [16].

This scenario could clearly be modeled using the fully compressible reacting flow equations, but the sound speed is high and the burners are large, and performing such a simulation at the resolution required for detailed characterization of the flame is computationally infeasible. Thus the goal of the work here is to construct a methodology in which the time scale at which the equations are evolved is that of the fluid velocity rather than the sound speed, but which can explicitly propagate the long-wavelength acoustic waves as they travel away from the flame and as they return and interact with the flame that created them.

This paper is the first of a series of papers describing the development of this methodology. For the purposes of this paper, one of the simplest low-Mach-number equation sets is considered, i.e. the variable density incompressible Euler equations. These equations allow different regions of the flow to have different densities, but do not allow any volumetric changes to occur (i.e. the material derivative of the density is zero). A hybrid approach is constructed in which variants of both the low-Mach-number equations and the fully compressible equations are solved in each time step; the computational efficiency of this approach results from the fact that the compressible equations are solved at a coarser resolution than the low-Mach-number equations. As a result, only long wavelength acoustic waves are resolved, yet the fine scale locally incompressible structure can still be resolved on the finer level(s).

The method is similar to the Multiple Pressure Variables (MPV) first introduced in a set of papers by Munz et al. [17–20]. The essence of the MPV approach is to decompose the pressure into three terms: the thermodynamic pressure p_0; the acoustic pressure p_1; and the perturbational pressure p_2. The acoustic signal is carried by p_1, and p_2 is used to satisfy the divergence constraint on the low-Mach-number levels and is defined as the solution to a Poisson equation. Different approaches for solving p_1 were proposed in the aforementioned references, for example by solving a set of Linearized Euler Equations (LEEs) on a grid that is a factor of $1/M$ coarser, where M is a measure of the Mach number of the flow. Differently, Peet and Lele [21] developed a hybrid method in which the exchange of information between the fully compressible and low-Mach-number regions occurs through the boundary conditions of overlapping meshes. The novelty of the present paper is that the fully compressible equations are solved without any approximation, and that an adaptive mesh refinement (AMR) framework is employed to optimize the performance of the algorithm. Thus, while the fully compressible equations are solved in the entire domain, with possible additional local refinement, the hybrid strategy developed in the present paper allows refined *patches* where the low-Mach-number equations are solved at finer resolution.

Note that there have been a number of other approaches to bridging the gap between fully compressible and low-Mach-number approaches. One alternative to the MPV methodology are the so-called *unified*, *all-speed*, *all-Mach* or *Mach-uniform* approaches [22–25], which consist of a single equation set that is valid from low to high Mach numbers. These methods retain the full compressible equation set, but numerically separate terms which represent convection at the fluid speed from acoustic effects traveling at the sound speed. Inherent in these approaches is that at least some part of the acoustic signal is solved for implicitly, which makes them inapplicable for our applications of interest in which explicit propagation of the long wavelength acoustic modes is preferred.

Note also that all of the methods described above involve feedback from the compressible solution to the low-Mach-number solution, and the reverse, thus they fundamentally differ from many hybrid methods employed in the aeroacoustics community, in which the acoustic calculation does not feed back into the low-Mach-number solution. Methods such as Expansion about Incompressible Flow (EIF) [26] can be used to calculate acoustic waves via Lighthill's analogy approach given an existing incompressible solution. A review of aeroacoustic methods is beyond the scope of this paper, but a comparison of EIF, MPV and LEEs is given in Roller et al. [27]. More recently, many groups [28–31] have investigated the coupling between a low-Mach-number detailed simulation of noise sources from a small scale turbulent flow, and the aeroacoustic propagation within a larger domain with the LEEs. It will be shown in the results section that the novel hybrid method developed in the present paper is able to tackle the same kind of problem while solving the purely compressible equations instead of the LEEs and allowing feedback of the acoustics into the low-Mach-number solution.

The remainder of this paper is organized as follows. In Section 2 the hybrid hierarchical grid strategy and governing equations that are solved at each resolution are presented. Then, in Section 3 the time advancement algorithm is detailed, as well as the procedures for interpolation and exchange of the variables between the different sets of equations at different levels. Finally, Section 4 contains the numerical results of the canonical test cases employed to assess the spatial and temporal rates of convergence of the hybrid method, as well as the simulation of the propagation of aeroacoustic waves