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Nonlinear pricing in linear cities with elastic demands

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ABSTRACT

Nonlinear road pricing charges each traveler based on his/her trip's corresponding particular attribute level. In order to help authorities in designing road pricing systems at a strategic level, this paper attempts to address two fundamental questions: (i) what is the value of pricing's nonlinearity for mitigating traffic congestion? (ii) if a nonlinear toll function is implemented, should it be convex, concave or other shape? Specifically, we consider distance-based pricing in linear cities. For linear monocentric cities with heterogeneous travelers, we show that the system optimal distance-based pricing indeed exhibits nonlinearity. It is proved that: (i) the cost-based system optimal toll function is monotonically increasing and concave with respect to the traveled distance; (ii) the time-based system optimal toll function always exists and is unique. If the initial proportion of each traveler group is invariant along a corridor and the demand function is of exponential type, then the time-based system optimal toll function enables the travelers, living further away from a city center, to face a lower toll level per unit distance. For a linear polycentric city, we demonstrate: (i) there always exists the system optimal differentiated (in terms of city centers) toll functions; (ii) it is highly possible that the system optimal non-differentiated toll function does not exist. Hence, we further propose an optimal toll design model, prove the Lipschitz continuity of its objective and adopt a global-optimization algorithm to solve it.

1. Introduction

Road pricing has been advocated to manage travel demand and relieve urban traffic congestion since 1920 (Pigou, 1920). Theoretically, the well-known first-best pricing principle assumes that all links on a transportation network are tollable, and the charging rate equals the user externality so that the optimal network traffic flow can be induced. However, the link-based marginal-cost pricing policy is a construct of little practical interest, because charging users on each network link will inevitably lead to the high operating cost and the difficulties of public acceptance (Ho et al., 2005). Practical implementation is of great importance for designing road pricing policy.

With the development of technologies in dedicated short communications, satellite systems and cellular networks (e.g., Dey et al., 2016), attribute-based road pricing has become more applicable and is receiving more attention. Basically, an attribute-based road pricing scheme charges users based on the quantity of their trips' corresponding particular traffic-related attribute, such as the distance, fuel consumption, emission and travel time. The advancements of vehicle-tracking and telecommunication technologies have technically enabled such an attribute-based road pricing scheme. For example, vehicle-to-infrastructure technologies along with an appropriate sensor deployment can support the recording of trip paths and travel time of vehicles (e.g., Zangui et al., 2013, 2015;

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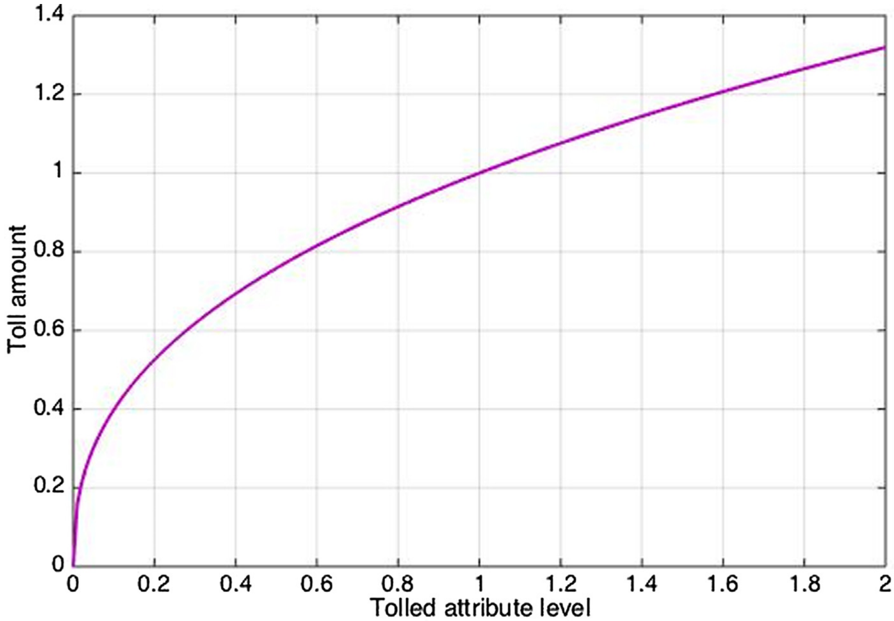

Fig. 1. Illustrative example of a nonlinear toll function.

Gentili and Mirchandani, 2018), thus facilitating the implementation of distance- or time-dependent pricing; the use of mobile internet usage data can further record the entire trip trajectories with a high resolution, allowing the management agencies to infer the travel distance, time, and emissions (e.g., Wang et al., 2018).

Under an attribute-based pricing scheme, for a traveler, a toll function, whose input is his/her trip's corresponding tolled attribute level, determines the specific charging rate. Essentially, attribute-based pricing not only avoids the difficulty of pricing individual links on dense urban networks but also considers the heterogeneity among trips (e.g., Liu et al., 2014a, 2014b; Daganzo and Lehe, 2015). In practice, transportation pricing is often nonlinear, i.e., the level of charged toll is not proportional to the amount of use (e.g., Wang et al., 2011), which leads to the concept of nonlinear pricing (e.g., Wilson, 1993). An illustrative example of a nonlinear toll function is shown in Fig. 1. For simplicity, we hereinafter use the term "nonlinear pricing" to represent attribute-based road pricing with a nonlinear toll function.

Depending on the choice of tolled attribute, nonlinear pricing could be distance toll (e.g., Lawphongpanich and Yin, 2012; Chen et al., 2016; Daganzo and Lehe, 2015; Liu et al., 2017), vehicle mileage fee[1] and gasoline tax (e.g., Mamun et al., 2016), joint distance and time toll (e.g., Liu et al., 2014b), and emission toll among others. Despite the widespread use of nonlinear road pricing, it is until recent years that scholars began to investigate the design of nonlinear tolling strategy. Wang et al. (2011) proposed a model to examine the effect of nonlinear pricing on a private road. Lawphongpanich and Yin (2012) formulated the network equilibrium with the piecewise linear distance-based toll, and proposed a coordinate search algorithm to determine the toll function which maximizes social welfare. Meng et al. (2013) investigated the problem of optimal distance-based toll under continuously distributed value of time, and Liu et al. (2014b) further investigated the joint distance and time toll on transportation networks, where the distance toll is based on a piecewise linear function. Daganzo and Lehe (2015) also investigated the problem of distance-based pricing for downtown zone with a setting of bottleneck model. Recently, Gu et al. (2018) further discussed the optimal distance- and time-dependent area-based pricing with a Network Fundamental Diagram approach.

However, all the studies above pre-assumed a specific form of nonlinear toll function and then solved bi-level programs to optimize the toll function parameters. There are two fundamental questions still remained to be addressed: (i) what is the value of pricing's nonlinearity for mitigating traffic congestion? (ii) if a nonlinear toll function is implemented, should it be convex, concave or other shape? Lin et al. (2018) were devoted to conducting sensitivity analyses for perturbation of toll function parameters on urban transportation networks, and they proposed a gradient sampling algorithm coupled with a branch-and-bound framework to globally solve a nonlinear toll function design problem on transportation networks. As observed in their numerical examples, the optimal toll function is concave in many scenarios. Unfortunately, the complexity of the two-dimensional space often makes it difficult to derive analytical properties to explain this phenomenon and better understand nonlinear pricing. In order to help authorities in designing road pricing systems at a strategic level, a simpler model is needed to enhance the analytical tractability.

[1] Note that vehicle mileage fee is different from distance toll in the sense that distance toll usually corresponds to a tolled area and charges travelers based on the distance travelled inside the tolled area whereas vehicle mileage fee is applicable to the whole network. As pointed out by Mamun et al. (2016), mitigating traffic congestion can be a "piggyback rider" to the vehicle mileage fee originally designed to recover the construction and operating costs of transportation infrastructure.