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Quadratic constrained weighted least-squares method for TDOA source localization in the presence of clock synchronization bias: Analysis and solution

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ABSTRACT

Time difference of arrival (TDOA) positioning is one of the widely used techniques for locating an emitter. Besides TDOA measurement errors, clock synchronization bias is an important factor that can degrade the localization accuracy. This paper focuses on TDOA localization using a set of receivers, where timing synchronization offsets exist among different receiver groups. A theoretical analysis is conducted and a new localization solution is developed for the case of imperfect time synchronization. The analysis starts with the Cramér–Rao bound (CRB) for the problem and derives the estimation bias and mean square error (MSE) using the classical quadratic constraint weighted least-squares (QCWLS) estimator, which does not consider synchronization errors. Additionally, an alternative performance measure, namely the localization success probability (SP), is introduced to evaluate the location accuracy. An explicit formula for calculating the localization success probability is presented. In addition, an improved quadratic constraint weighted least-squares estimator that accounts for synchronization errors is proposed to reduce the positioning errors. The Lagrange multiplier technique is used to solve this estimator. As a byproduct, a closed-form solution for the estimation of clock bias is also provided. First-order perturbation analysis reveals that the performance of the proposed estimates achieves the Cramér–Rao bound. Simulations corroborate the theoretical results and the good performance of the proposed method.

1. Introduction

The problem of passive source location has received considerable interest in recent decades within many disciplines, such as signal processing, wireless communications, passive radar, vehicular techniques, and underwater acoustics. Most localization methods involve two processing steps. In the first step, intermediate parameters that are embedded in the received signals are estimated. These parameters can be characterized by the emitter's position and are usually the angle of arrival (AOA) [1,2], time difference of arrival (TDOA) [3–11], time of arrival (TOA) [12,13], frequency difference of arrival (FDOA) [14–18], frequency of arrival (FOA) [19], received signal strength (RSS) [20,21], and gain ratios of arrival (GROA) [22,23]. In the second step, the previously estimated parameters are used to determine the source location. This two-step procedure is also known as the decentralized approach [24]. For a stationary emitter, TDOA positioning is one of the most widely used schemes. In contrast to time-of-arrival positioning, it does not require knowledge of the time that the received signal was transmitted from the transmitter and is suitable for passive location. Moreover, it has superior localization performance compared with other measurements, such as AOA and RSS in general [10].

In this work, we consider the localization of a single stationary source using TDOA measurements made with a number of spatially separated receivers. Over the years, much literature has been published on source localization using TDOAs. Indeed, TDOA localization is not a trivial task owing to the nonlinear nature of the estimation problem. Consequently, many methods are iterative; e.g., the Taylor-series (TS) linearization method [7,11], constrained total-least-squares (CTLS) method [10], and steepest-descent method [25]. Iterative methods generally provide high accuracy at reasonable noise levels but they are computationally intensive because of their iterative course. Moreover, they require proper initial solutions for emitter positions, which are not easily obtained in practice. Note that the convex optimization method [9]

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is also implemented via numerical optimization techniques, such as the interior-point method [26], but it has no local minima or saddle points and thus convergence to the global minimum can be guaranteed. An alternative for source location is closed-form solution methods, which are attractive because they do not require initial position guesses and do not face the divergence problem faced by the iterative technique. The most representative closed-form method is the two-step weighted least-squares (TWLS) minimization algorithm [3,8,15,16], which is implemented by solving two pseudo-linear equations sequentially in the weighted-least-squares (WLS) sense. Although the TWLS approach is more computationally efficient, it usually has a lower noise endurance threshold than the iterative methods, which means that the solution can reach Cramér–Rao bound (CRB) accuracy only at a low noise level.

Besides the above-mentioned localization approaches, there is another method that requires the estimation of Lagrange multipliers and is used for source localization. This method is called the quadratic constrained weighted least-squares (QCWLS) method [4,5,13,14,17,18,27] because there are quadratic equality constraints in the corresponding optimization model. It is noteworthy that compared with the aforementioned iterative methods, the QCWLS method has less computational burden because the dimension of Lagrange multipliers is generally smaller than that of the emitter position parameter. Moreover, the QCWLS method more easily avoids divergence because the initial guess for Lagrange multipliers is readily available. Meanwhile, compared with the closed-form methods (e.g., the TWLS method), the QCWLS method can tolerate a higher noise level before the thresholding effect occurs. In this paper, we focus on the QCWLS method owing to its superior performance.

In addition to TDOA measurement errors, another major factor affecting positioning accuracy is the time synchronization errors of the receivers. Location methods must therefore consider clock bias. Indeed, much attention has been paid to the problem of joint synchronization and localization in recent years. Moreover, most of these studies focus on sensor localization in a wireless sensor network (WSN) [28–37]. In a wireless sensor network, the sensor nodes consist of both anchor nodes, whose locations are known, and source nodes, whose locations must be estimated. Generally, it may be reasonable to assume that the clocks of the anchor nodes are synchronized with each other, but it is not reasonable to assume that the source nodes are synchronized with each other or with the anchor nodes. Messages are exchanged among anchor nodes and source nodes in one-way or two-way transmission to jointly estimate the locations of the source nodes and the clock parameters (i.e., time skew and time offset). Meanwhile, there is related work in the field of passive localization. Specifically, the methods for joint time synchronization and source localization using time-of-arrival measurements have been presented [38–43]. In addition, a high-accuracy TDOA-based technique without time synchronization has been designed [44]. Note that although the time synchronization requirement of this method can be removed employing a two-signal sensing and sample counting technique, a successive signal must be produced artificially. An alternative efficient TDOA-based localization algorithm without synchronization between base stations has been proposed [45]. However, this method must vary the locations of the source and, hence, it cannot be applied to locate a stationary source. Additionally, a TDOA-based localization approach that simultaneously estimates the source position and clock offset has been given [46]. This method is implemented using the TS iteration technique, which requires good initialization and has high complexity.

It is worth pointing out that asynchronous sampling does not always occur for different receivers. Indeed, when receivers are close to each other, it is relatively simple to achieve synchronous sampling through the use of a single hardware device with multichannel acquisition capabilities. Hence, synchronization errors should be considered only when receivers well separated. A number of spatially separated receivers for TDOA localization have been categorized into many groups and synchronized within each group with synchronization timing offsets among different groups [47]. On the basis of this localization scenario, an algebraic closed-form solution of the emitter location, receiver positions, and synchronization offsets has been developed [47]. Although the performance of this method was shown to reach CRB accuracy analytically, the method may suffer from a thresholding effect at a low noise level. The present paper assumes that the localization scenario is similar to that in [47]. A theoretical analysis is performed and a new localization solution is presented for the case of non-ideal time synchronization. We first derive the estimation bias and mean square error (MSE) in the source location estimate when an estimator assumes that there are no synchronization errors but in fact there are. The estimator used for performance analysis is the previously introduced QCWLS method [4,5,13,14,17,18,27], which does not take synchronization errors into account. In addition to the estimation bias and MSE, an alternative performance measure is introduced to evaluate the location accuracy; i.e., the localization success probability (SP). An exact formula for computing the localization SP of the QCWLS estimator is provided. Here, the effects of synchronization errors on the classical QCWLS localization method are studied for the first time to the best knowledge of the authors. Subsequently, a new QCWLS method that accounts for clock synchronization bias is developed to improve the localization accuracy. The Lagrange multiplier technique is used to solve the proposed QCWLS estimator. As a byproduct, a closed-form solution for the estimation of clock offset is provided. The proposed solutions are proved theoretically to reach CRB accuracy under the Gaussian data model. The novelty and technical contributions of the paper are summarized as follows.

- In the presence of clock synchronization bias, we derive compact expressions for the estimation bias and MSE of the classical QCWLS method, which does not consider synchronization errors.
- We introduce an alternative performance measure, namely localization SP, and derive an explicit formula for computing the localization SP of the classical QCWLS estimator when synchronization errors are present.
- We propose a new QCWLS estimator for TDOA positioning in the presence of clock synchronization bias. Although the QCWLS method has been proposed for the source location problem, it is not directly applicable to the scenario that synchronization errors exist. Additionally, a closed-form solution for the estimation of clock offset is provided.
- The theoretical performance of the proposed QCWLS method is derived and we show analytically that the new method attains the corresponding CRB for both the source position and clock bias.
- As a byproduct of our theoretical derivation, we present a new method of determining the radius of circular error probable (CEP) for any unbiased localization algorithm. This method is suitable for not only two-dimensional (2-D) localization but also the three-dimensional (3-D) scenario.
- In contrast to some existing analysis approaches, our theoretical analysis is carried out with an equality constraint and makes use of the property of the orthogonal projection matrix. More importantly, it is performed in a general mathematical framework, which is not limited to a specific signal metric.

The remainder of this paper is organized as follows. In Section 2, the localization scenario is described and the measurement model is formulated. Section 3 derives the estimation bias, MSE