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Justification of Stokes Hypothesis and Navier-Stokes First Exact Transformation for Viscous Compressible Fluid

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Abstract In this paper Stokes' hypothesis receives the physical justification and alternative **mathematical** formulation. An alternative formulation is more general and easily checked. As well as in linear elasticity, the pressure in Navier-Stokes equations can be eliminated as unknown function. To do this transformation it is necessary **to** use the well known derivation analogies of Navier-Cauchy (also called Lame's) equations. In this case the well known problem of "second viscosity" disappears. Thus the fundamental defining relation for pressure is no **longer necessary** and the Navier-Stokes and Lame's equations **exactly** coincide. The Navier-Stokes equations are not physically exact because the traditional assumption **of const bulk viscosity is not correc**t. The analysis presented here corrects this error and more exact Navier-Stokes equations have been proposed. This paper written in a way that gives insight to mathematicians, physicists, engineers and students who may not be experts in this topic.

Keywords Bulk viscosity, Second viscosity, Poisson's ratio, Navier-Stokes equation, Cauchy momentum equation, Linear elasticity

1. Introduction

1.1. Fundamental and Basic Equations in Linear Elasticity and in Hydromechanics

For derivation of the classical equations both in linear elasticity and in hydromechanics the **same fundamental equations and analogous basic equations** are used. These **fundamental** equations **(for any deformable media)** in a vector form **are** called the Cauchy momentum equations and can be written as

$$\rho\vec{F} + \operatorname{div} T_\sigma = \rho\ddot{\vec{u}} \ . \qquad (1)$$

Here $\vec{F}$ - vector of unit mass force, T_σ - Cauchy stress tensor, $\ddot{\vec{u}} = \frac{d\dot{\vec{u}}}{dt}$ - acceleration vector, $\dot{\vec{u}}$ -velocity vector, ρ - density.

The expanded form (1) is shown in [1, p. 60].

The basic equations in linear elasticity as can be seen in [1, p. 64-65; 2, p. 144; 3, p. 255] are

$$D_\sigma = 2GD_\varepsilon \ . \qquad (2)$$

Here D_σ -deviatoric stress tensor, D_ε -deviatoric deformation tensor, G - shear modulus.

In the component form basic equations (2) can be written so

$$\sigma_{ii} - \sigma_o = 2G(\varepsilon_{ii} - \varepsilon_o),\ \ \sigma_{ij} = 2G\varepsilon_{ij},\ \ i = x, y, z,$$
$$\varepsilon_o = \frac{1}{3}(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}),\ \ \sigma_o = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}). \qquad (2,a)$$

Here $\sigma_{ii} - \sigma_o$ - components of deviatoric stress tensor, $\dot{\varepsilon}_{ii} - \dot{\varepsilon}_o$ - components of deviatoric deformations tensor, σ_{ii} - normal stresses, σ_{ij} - shear stresses, $\varepsilon_{ii} = \frac{\partial u_i}{\partial x_i}$, $\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$- components of deformations tensor (normal and shear).

For viscous compressible fluid we **use fundamental** Cauchy momentum equation (1) and identical **basic** equations (2) and (2,a) **by replacing** the $G \to \mu,\ \varepsilon_{ii} \to \dot{\varepsilon}_{ii},\ \varepsilon_{ij} \to \dot{\varepsilon}_{ij},\ \varepsilon_o \to \dot{\varepsilon}_o,\ u_i \to \dot{u}_i$

$$D_\sigma = 2\mu D_{\dot{\varepsilon}} \ . \qquad (2,b)$$

$$\sigma_{ii} - \sigma_o = 2\mu(\dot{\varepsilon}_{ii} - \dot{\varepsilon}_o),\ \ \sigma_{ij} = 2\mu\dot{\varepsilon}_{ij},\ \ i = x, y, z,$$
$$\dot{\varepsilon}_o = \frac{1}{3}(\dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} + \dot{\varepsilon}_{zz}) = \frac{1}{3}\operatorname{div}\dot{\vec{u}}, \qquad (2,c)$$
$$\sigma_o = -p = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}).$$

Here $D_{\dot{\varepsilon}}$ -deviatoric of deformation rate tensor (DDRT), p - pressure, $\dot{\varepsilon}_{ii} - \dot{\varepsilon}_o$ - components of DDRT, $\dot{\varepsilon}_{ii} = \frac{\partial \dot{u}_i}{\partial x_i}$, $\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial \dot{u}_i}{\partial x_j} + \frac{\partial \dot{u}_j}{\partial x_i}\right)$- components of deformation rate tensor, μ - dynamic viscosity.