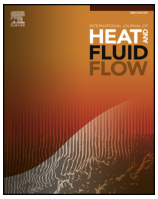
International Journal of Heat and Fluid Flow 73 (2018) 258–269

Contents lists available at ScienceDirect

International Journal of Heat and Fluid Flow

journal homepage: www.elsevier.com/locate/ijhff



Analytical wall function including roughness corrections

F. Chedevergne

ONERA, The French Aerospace Lab, Toulouse F-31055, France

ARTICLE INFO

Keywords:
Analytical wall function
Heat transfer
Turbulence modeling
Wall roughness

ABSTRACT

Inspired by the work of Aupoix (2015a,b) and relying on the analytical wall function of Suga et al. (2006), this paper proposes a modified version of the wall model capable of accounting for roughness effects. The thermal correction was enhanced to capture roughness effects due to the increase of the wetted surface of the walls. A derivation of the model adapted to configurations with very large roughness is also proposed. The new model is compared to the former analytical wall function formulation using several rough configurations for which experimental data are available. The validation test cases have been chosen to highlight the improvements brought by the present work.

1. Introduction

Investigations on effects of roughness on boundary layers date back to the early1930's with major contributions by Nikuradse (1933, 1937). Roughness is known to increase wall friction and heat transfer. In the full rough regime, the surface friction increase is related to the pressure drag induced by the presence of rough elements on walls. As for heat transfer, combined effects of increased near wall turbulence levels and increased wetted surface are responsible of the rise. Owing to the distinct types of mechanisms involved in the friction and heat transfer increases, wall roughness cause less heat transfer growth than friction growth rendering the Reynolds analogy no longer valid on rough walls.

Even though a large and increasing number of industrial applications (wear of the blades in turbines, icing accretion on wings, ablation on reentry vehicles, etc.) require to account for roughness effects, most of the efforts in the fluid mechanics community concentrate on modeling boundary layers over smooth surfaces. In recent years, progress has been made in the development of turbulence models capable of reproducing roughness effects and applicable in an industrial context. Among methods used to model rough flows, the equivalent sand grain method revealed to be the most appropriate to engineering applications. The method is made of two steps. First, using correlations the rough surface is reduced to an equivalent surface covered by sand grains of height h_s that will produce the same friction increase. Then, an ad-hoc correction driven by h_s is made on a turbulence model in order to artificially enhance turbulence in the wall region and reproduce the friction increase. On this basis, Aupoix (2015b) built a correction for the SST $k-\omega$ turbulence model of Menter (1994). This first correction has been completed with an additional correction (Aupoix, 2015a) to account for thermal effects due to roughness, since the Reynolds analogy cannot be used.

Combined, both corrections work remarkably on a variety of heated boundary layers over rough walls. But such an approach requires the use of fine meshes at walls inasmuch as it is a low-Reynolds number (LRN) approach. For some industrial applications, there is a crucial need to reduce CPU costs while making use of coarse meshes. For instance, in the framework of the development of ONERA's icing suites IGLOO2D (Villedieu et al., 2014; Trontin et al., 2017) and IGLOO3D (Radenac, 2016), which involve a large number of cycles of a flow solver, different solutions are currently being studied to avoid having resource to the use of wall-refined meshes. The first option consists in coupling a boundary layer code (Bayeux et al., 2017; Bempedelis et al., 2017) to an Euler solver; the second being the development of a wall function approach in a Navier–Stokes environment.

Starting from the analytical wall function (AWF) developed by Craft et al. (2002) which permits the computation of wall flows while including many physical effects (buoyancy, laminarization, properties temperature dependency), Suga et al. (2006) introduced modifications to account for rough walls. A simplified version of the original AWF by Craft et al. was used in Suga's work which do not include all refinements. Although very satisfactory results were obtained by Suga et al. some points deserve further improvements. More specifically, Suga's model has been validated on applications where roughness effects where limited, *i.e.* for which h_s values were not too large. In addition, wetted effects that are known to play a role in the heat transfer over rough walls are not accounted for in Suga's model. Therefore, the present study investigates the possibility of matching the work by Suga et al. with the corrections initiated by Aupoix to extend the validity range of the AWF. First a reminder of the principles driving Aupoix's corrections is made. Subsequently, the AWF formulation of Suga et al. is recapped. Finally, a new proposal is formulated and

---

E-mail address: francois.chedevergne@onera.fr.

https://doi.org/10.1016/j.ijheatfluidflow.2018.08.001
Received 31 October 2017; Received in revised form 28 June 2018; Accepted 1 August 2018

validation cases highlighting the strengths of the present AWF are performed.

2. Aupoix's roughness corrections

Only distributed roughness will be considered, *i.e.* where characteristic lengths (height, span and spacing) are small compared to boundary layer thickness. Additionally, only k-type roughness (Perry et al., 1969), for which effects are related to their heights, will be treated in the following study. Relying on the classical description of turbulent boundary layer over rough wall initiated by Nikuradse (1933, 1937) and previous works (Aupoix and Spalart, 2003; Knopp et al., 2009), Aupoix developed a dynamic correction (Aupoix, 2015b) recreating the friction increase due to roughness in the framework of the $k-\omega$ SST model (Menter, 1994). Then, since the Reynolds analogy no longer holds for turbulent boundary layer over rough walls, Aupoix enhanced his work with a thermal correction (Aupoix, 2015a). Both aspects are briefly reminded hereinafter.

2.1. Dynamic correction

Nikuradse (1933, 1937) pointed out that above roughness the logarithmic law is preserved but shifted. The velocity profile in wall variables u^+ reads:

$$u^+ = \frac{1}{\kappa}\ln y^+ + C - \Delta u^+ \qquad u^+ = \frac{u}{u_\tau}, \quad y^+ = \frac{yu_\tau}{\nu} \tag{1}$$

where $C = 5.5$ and κ is the Kármán constant. Note that Nikuradse (1933, 1937) retained $\kappa = 0.4$ in his studies. However, in the present study the classical value $\kappa = 0.41$ is used.

The friction velocity used to defined wall quantities in all the following is $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$, τ_w being the wall shear stress. In Nikuradse's experiments, τ_w is determined from the measured pressure drop in a pipe. The shift Δu^+ is related to the equivalent sand grain height $h_s^+ = \frac{h_s u_\tau}{\nu}$. Multiple expressions have been derived in the literature and in the present work the compact form provided by Grigson (1992) and obtained from Colebrook's data (Colebrook and White, 1937; Colebrook, 1938–1939) is retained:

$$\Delta u^+ = \frac{1}{\kappa}\ln\left(1 + \frac{h_s^+}{\exp(3.25\kappa)}\right) \tag{2}$$

Starting from this fundamental observation on boundary layer profiles, Aupoix and Spalart (2003) proposed a general strategy to reproduce the shift in turbulence models while artificially increasing turbulent viscosity μ_t at the wall. The leading principles are the following: first, a wall shift y_0 is introduced so that velocity gradients over rough and smooth surfaces satisfy:

$$\left.\frac{\partial u_r^+}{\partial y^+}\right|_{y^+} = \left.\frac{\partial u_s^+}{\partial y^+}\right|_{y^+ + y_0^+} \tag{3}$$

where subscripts r and s refer to rough and smooth surfaces respectively. After integration along the wall normal it yields:

$$u_r^+(y^+) = u_s^+(y^+ + y_0^+) - u_s^+(y_0^+) \tag{4}$$

The shift Δu^+ is thus directly given by:

$$\Delta u^+ = u_s^+(y_0^+) \tag{5}$$

In turbulent boundary layers, considering a constant total shear, the momentum equation reduces to:

$$(1 + \mu_t^+)\frac{\partial u^+}{\partial y^+} = 1 \tag{6}$$

with $\mu_t^+ = \frac{\mu_t}{\mu}$.

Finally, from Eq. (3) it becomes:

$$\mu_{t_r}^+(y^+) = \mu_{t_s}^+(y^+ + y_0^+) \tag{7}$$

The initial search for the Δu^+ shift has been transferred to the one for y_0^+ that yields the desired eddy viscosity increase. In particular at the wall, $\mu_{t_{rw}}$ may not be zero:

$$\mu_{t_{rw}}^+ = \mu_{t_s}^+(y_0^+) \tag{8}$$

In practice, once a relationship between Δu^+ and h_s^+ such as in Eq. (2) is known, a smooth profile expression can be used to find the y_0^+ that satisfies Eq. (5). Then, considering the $k-\omega$ SST turbulence model, it is straightforward to get the $k_s^+(y_0^+)$ and $\omega_s^+(y_0^+)$ that yield the k_w^+ and values to be imposed at the wall to recover Eq. (8). Turbulent scalars are made dimensionless with the friction velocity u_τ and the viscosity ν such that $k^+ = \frac{k}{u_\tau^2}$ and $\omega^+ = \frac{\omega\nu}{u_\tau^2}$. Expressions $k_w^+(h_s^+)$ and $\omega_w^+(h_s^+)$ have been obtained by Aupoix (2015b) and read:

$$\begin{aligned}
k_w^+ &= \max(0;\, k_0^+)\\
k_0^+ &= \frac{1}{\sqrt{\beta^*}}\tanh\left[\left(\frac{\ln\frac{h_s^+}{30}}{\ln 10} + 1 - \tanh\frac{h_s^+}{125}\right)\tanh\left(\frac{h_s^+}{125}\right)\right]\\
\omega_w^+ &= \frac{300}{h_s^{+2}}\left(\tanh\frac{15}{4h_s}\right)^{-1} + \frac{191}{h_s^+}\left[1 - \exp\left(-\frac{h_s^+}{250}\right)\right]
\end{aligned} \tag{9}$$

2.2. Thermal correction

The dynamic correction is an ad-hoc correction that reproduces the pressure effect on drag through an increase of the eddy viscosity at the wall. If no further correction is introduced in the $k-\omega$ SST turbulence model, the eddy thermal conductivity at the wall $\lambda_{t_{rw}}$ is overestimated since the Reynolds analogy is no longer valid for rough surfaces. Using the dynamic correction (9) associated with a constant turbulent Prandtl number Pr_t this approach was proven to overestimate heat transfers (Aupoix, 2015a). A simple way to derive a thermal correction is to modify the turbulent Prandtl number by writing:

$$Pr_t = Pr_{t_\infty} + \Delta Pr_t \tag{10}$$

where Pr_{t_∞} is the standard turbulent Prandtl number value 0.9. Several parameters rule the correction developed by Aupoix (2015a), each one representing different physical behaviors. The thermal correction ΔPr_t must be restricted to a certain extent from the wall, *i.e.* within the roughness sublayer. The latter corresponds to the near wall region where the flow is strongly affected by the presence of roughness. Above the roughness sublayer, the flow is locally homogeneous in planes parallel to the wall. The roughness sublayer generally expands up to three to five times the mean roughness height h. An exponential decay involving this mean roughness height h has been introduced into ΔPr_t to limit the extent of the correction. The equivalent sand grain height h_s, which provides the velocity shift Δu^+, allows to account for the turbulence diffusion, whereas wetted surface effects are accounted for using an additional parameter S_{corr}. The latter is the corrected wetted surface ratio defined using the surface geometry where troughs below the reference (melt-down surface) are neglected (Aupoix, 2015a). The final correction of Aupoix reads as follow:

$$\begin{aligned}
\Delta Pr_t &= \mathcal{F}^A \exp(-y/h)\\
\mathcal{F}^A &= A\Delta u^{+2} + B\Delta u^+\\
A &= (0.0155 - 0.0035S_{corr})[1 - \exp(-12(S_{corr} - 1))]\\
B &= -0.08 + 0.25\exp(-10(S_{corr} - 1))\\
\Delta u^+ &= \Delta u^+(h_s^+)
\end{aligned} \tag{11}$$

In the studied subsonic heated boundary layers configurations, ΔPr_t lies between 0 and 2 as shown on Fig. 3.