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On modeling and eddy-resolving simulations of flow, turbulence, mixing and heat transfer of electrically conducting and magnetizing fluids: A review

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ABSTRACT

The paper provides a concise review of previous and current trends in modeling and simulations of flow and heat transfer of electrically conducting and magnetizing fluids. For the electrically conducting fluids, the focus is on modeling of the interactions between velocity fluctuations and imposed magnetic field (magnetohydrodynamic (MHD) turbulence). We address the most commonly used approaches to simulate the MHD turbulent flows: the single-point Reynolds-Averaged Navier–Stokes (RANS) and eddy-resolving approach. The former includes the second-moment closures with additional terms due to the presence of the Lorentz force and its eddy-viscosity based simplifications. The latter covers direct numerical resolving (DNS), large-eddy (LES) and very-large eddy (VLES) simulation methods, and their verification with available experiments. Within this framework, we present some of our previous and current achievements dealing with applications of the (electro)magnetic control of flow, heat transfer and mixing. Application examples are selected to cover a wide range of interacting parameters for both electrically conducting and magnetizing fluids, in laminar, transient and fully developed turbulent regimes (including also one- and two-way coupled MHD phenomena).

1. Introduction

Examples of flow of electrically conducting and magnetized fluids include a large number of situations in technological and industrial applications (e.g. continuous casting of liquid steel, crystal growth, arc welding, aluminium reduction cells, liquid metal blankets of the newest generation of fusion reactors, magnetohydrodynamic (MHD) pumps, flow-meters, generators, propulsion devices, etc., Branover (1978); Moffatt (1978), Sutton and Sherman (1993); Davidson (2001); Molokov et al. (2007)), astrophysical (e.g. accretion disks, galaxy clusters, solar winds, sunspot, etc, Rüdiger and Hollerbach (2004); Borrero and Ichimoto (2011)) and geophysical (e.g. origins of the Earth's magnetic field, etc., Krause and Rädler (1980); Gailitis (2004)) applications. In the present paper, we focus mainly on the industrial and technological applications for which the one-way coupling between the imposed magnetic field and underlying flow is assumed Knaepen and Moreau (2008). One of the examples also includes the two-way coupled MHD phenomena - of importance for geophysical applications - to mimic experiments designed to confirm the magnetic-dynamo concept behind the origin of the Earth's magnetic field (e.g. the Riga-dynamo experiment). For MHD applications, we consider weakly (with the electric conductivity of $\sigma = O(1)$ S/m) and strongly ($\sigma = O(10^6)$ S/m) conducting fluids, Table 1. For the first category, it is necessary to impose a combination of external magnetic and electrical fields that are able to generate sufficiently strong local Lorentz forces that they impact the underlying fluid flow. We will apply this forcing for cases where the Lorentz force needs to be distributed in the proximity of the wall to affect the hydrodynamic and thermal boundary layers. For the second category of strongly electrically conducting fluids, application of an external magnetic field is sufficient by itself to generate a significant Lorentz force. Note that in such a case, the imposed magnetic field in combination with underlying fluid motion generates the electric current which in turn interacts with the imposed magnetic field to finally produce the Lorentz force.

In addition to electrically conducting fluids, in this review, we also address magnetizing fluids. Such fluids are magnetically neutral until entering a magnetic field gradient. Then, they can be attracted (paramagnetic) or repulsed (diamagnetic) by the imposed magnetic field gradients. Note that for such fluids, there is no electric current generated. Furthermore, due to their relatively low values of magnetic susceptibility ($|\chi| = O(10^{-5} - 10^{-6})$, Table 2), one has to apply rather strong magnetic field gradients (Table 3) to produce sufficiently strong magnetization (Kelvin) forces. The latest generation of superconducting magnets (with their strengths even up to 5–30 T), made it possible to

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Table 1
Electrical conductivity (σ in [S/m]) of some materials.

	Seawater	Air	Mercury	Stainless steel	Blood
Electrical conductivity	4.8	$3 \cdot 10^{-15}$	$1.02 \cdot 10^{6}$	$1.45 \cdot 10^{6}$	10–20
Ref.	Janz and Singer (1975)	Pawar et al. (2009)	Mott (1966)	Ralls et al. (1976)	Hirsch et al. (1950)

Table 2
Magnetic susceptibility (χ in [-]) of some materials.

	Water	Air	Bismuth	Blood (oxygenated)
Magnetic susceptibility	$-9.035 \cdot 10^{-6}$	$3.6 \cdot 10^{-7}$	$-1.66 \cdot 10^{-4}$	$-9.15 \cdot 10^{-6}$
Ref.	Arrighini et al. (1968)	Schenk (1993)	Otake et al. (1980)	Spees et al. (2001)

Table 3
Some example of magnetic fields (in [T]).

	Earth	Sunspots	MRI scanner	NdFeB magnet
Magnetic field strength	$(2.5-6.5) \times 10^{-4}$	0.3	$1.5-7$	1.25
Ref.	Finlay (2010)	Borrero and Ichimoto (2011)	Van Elderen et al. (2009)	Jakob (2010)

Table 4
Specification of additional dimensionless magnetic terms in $\overline{v_i' v_j'}$ equations for a fully developed plane channel flow under presence of a transverse magnetic field, $S_{ij}^{\mathrm{L}} = S_{ij}^{\mathrm{L1}} + S_{ij}^{\mathrm{L2}}$.

	$\overline{u'u'}$	$\overline{v'v'}$	$\overline{w'w'}$	$\overline{u'v'}$
S^{L1}	$2\frac{\mathrm{Ha}^2}{\mathrm{Re}}B_y^+ \overline{u'^+\frac{\partial \phi'^+}{\partial z^+}}$	0	$-2\frac{\mathrm{Ha}^2}{\mathrm{Re}}B_y^+\overline{w'^+\frac{\partial \phi'^+}{\partial x^+}}$	$\frac{\mathrm{Ha}^2}{\mathrm{Re}}B_y^+\overline{v'^+\frac{\partial \phi'^+}{\partial z^+}}$
S^{L2}	$-2\frac{\mathrm{Ha}^2}{\mathrm{Re}}B_y^+B_y^+ \overline{u'^+u'^+}$	0	$-2\frac{\mathrm{Ha}^2}{\mathrm{Re}}B_y^+B_y^+ \overline{w'^+w'^+}$	$-\frac{\mathrm{Ha}^2}{\mathrm{Re}}B_y^+B_y^+ \overline{u'^+v'^+}$

Table 5
Specification of additional dimensionless magnetic terms in k and ε equations for a fully developed plane channel flow under a transverse magnetic field, $S_\phi^{\mathrm{L}} = S_\phi^{\mathrm{L1}} + S_\phi^{\mathrm{L2}}$.

	k	ε
S^{L1}	$\frac{\mathrm{Ha}^2}{\mathrm{Re}}B_y^+\left(\overline{u'^+\frac{\partial \phi'^+}{\partial z^+}} - \overline{w'^+\frac{\partial \phi'^+}{\partial x^+}}\right)$	$2\frac{\mathrm{Ha}^2}{\mathrm{Re}}B_y^+\left(\overline{\frac{\partial u'^+}{\partial x_l^+}\frac{\partial^2 \phi'^+}{\partial x_l^+ \partial z^+}} - \overline{\frac{\partial w'^+}{\partial x_l^+}\frac{\partial^2 \phi'^+}{\partial x_l^+ \partial x^+}}\right)$
S^{L2}	$-\frac{\mathrm{Ha}^2}{\mathrm{Re}}B_y^+B_y^+(\overline{u'^+u'^+} + \overline{w'^+w'^+})$	$-2\frac{\mathrm{Ha}^2}{\mathrm{Re}}B_y^+B_y^+\left(\overline{\frac{\partial u'^+}{\partial x_l^+}\frac{\partial u'^+}{\partial x_l^+}} + \overline{\frac{\partial w'^+}{\partial x_l^+}\frac{\partial w'^+}{\partial x_l^+}}\right)$

produce sufficiently strong magnetization forces even for ordinary fluids such as air or water (Tagawa et al., 2002; 2003; Ozoe, 2005). This paved the way to perform some fundamental studies dealing with combined effects of thermal buoyancy and magnetization, and to analyze the fully developed turbulent flow regimes for the first time.

We start from the basic governing equations in fluid mechanics (hydrodynamics) and electromagnetism (electrodynamics), and their coupling through the Lorentz (for the MHD) or Kelvin (magnetization) forces. Characteristic non-dimensional parameters are introduced and used to classify different categories of fluid flow and (electro)magnetic field interactions. The basic framework of the modeling of MHD turbulence is introduced and presented for both the second-moment and eddy-viscosity based enclosure. A concise overview of past efforts and the present state of turbulence models is presented. This is followed by the eddy-resolving simulations: DNS and (V)LES. Some of the generic cases are addressed, including a decay of initially isotropic homogeneous turbulence subjected to a uniform magnetic field, the turbulent channel, pipe, and duct flows as well as thermal-buoyancy driven flows - all subjected to a uniform magnetic field of different orientation and strength. Finally, we address in more detail selected cases of various applications, which cover an extended range of working (laminar, transient and turbulent flows) and interacting (weak- or strong-coupling) regimes - in both MHD and thermo-magnetic types of phenomena. We complete our review with some possible routes of future investigation dealing with various aspects of theoretical, numerical simulation and experimental studies.

2. Governing equations

Here we will provide a short overview of the main equations describing the behavior of electrically conducting or magnetizing fluids in the presence of an externally imposed or internally induced electric and magnetic field(s). The link between equations of hydrodynamics and electrodynamics is established through the presence of the Lorentz and magnetization forces acting on the fluid.

2.1. Hydrodynamics

The conservation of mass, momentum and thermal energy constitute the closed system of equations describing the instantaneous fluid motion, and are written as:

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \nu \nabla^2 \mathbf{v} + \mathbf{f}^{\mathrm{P}} + \mathbf{f}^{\mathrm{B}} + \mathbf{f}^{\mathrm{L}} + \mathbf{f}^{\mathrm{M}} \tag{2}$$

where 'ν' is the kinematic viscosity of fluid. The body forces included in the right-hand-side of the momentum equation represent the pressure ($\mathbf{f}^{\mathrm{P}}$), the thermal buoyancy ($\mathbf{f}^{\mathrm{B}}$), the Lorentz ($\mathbf{f}^{\mathrm{L}}$), and the magnetization ($\mathbf{f}^{\mathrm{M}}$) forces, respectively. These forces are calculated as follows:

$$\mathbf{f}^{\mathrm{P}} = -\frac{1}{\rho_0}\nabla p, \quad \mathbf{f}^{\mathrm{B}} = \left(\frac{\rho - \rho_0}{\rho_0}\right)\mathbf{g}, \qquad \mathbf{f}^{\mathrm{L}} = \frac{1}{\rho_0}(\mathbf{j} \times \mathbf{b}), \quad \mathbf{f}^{\mathrm{M}} = \mu_0 \mathbf{m} \cdot \nabla \mathbf{h} \tag{3}$$

where 'ρ_0' is the fluid density at the reference temperature (which is defined as $\theta_0 = (\theta_h + \theta_c)/2$), the '$p$' is the pressure, the 'ρ' is the fluid density at a temperature (θ), '$\mathbf{g}$' is the gravitational vector, the '$\mathbf{j}$' is the total electrical current density, the '$\mathbf{b}$' is the total magnetic field, the 'μ_0' is the magnetic constant, the '$\mathbf{m}$ is the magnetization, and finally, the '$\mathbf{h}$' is the external magnetic field.