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Radiative heat transfer in an absorbing-emitting semi-transparent medium enclosed in a two-dimensional parallelogram cavity with diffuse reflecting boundaries

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A B S T R A C T

An exact analytical description of the internal radiative field inside an emitting-absorbing gray semi-transparent medium at radiative equilibrium, enclosed in a two-dimensional parallelogram cavity bounded by diffuse reflecting surfaces is proposed. The exact expressions of the incident radiation and the radiative flux field are angularly and spatially discretized with a double Gauss quadrature, and the temperature field is obtained in an iterative way. Some examples show that the proposed method gives perfectly smooth results of good accuracy for both the temperature and flux fields inside the medium.

1. Introduction

Necessity to take into account the radiative effects in a large class of thermal problems involving high temperature complex devices has enhanced in the last decades the development of several numerical techniques applied in various geometries. The case of the parallelogrammic cavity, which has some applications in solar energy systems, has been examined in natural convection studies [1–4], also coupled with magnetic effects [5]. Convection inside open or closed rectangular cavities has been studied when taking into account the presence of isothermal diffusely reflecting walls, so as active boundaries subjected to conditions of imposed flux [6], which shows complex interactions with wall radiation. Gonzales et al. [7] examined the case of an open rectangular cavity with two adiabatic walls and an isothermal one, and shown that surface radiation altered substantially the basic flow pattern so as the overall thermal performance. Due to the presence of non-perpendicular surfaces, the convection in a parallelogrammic device is slightly different from the one described in square cavity, with some spectacular effects on the temperature field inside the cavity, depending on the angle between two adjacent boundaries. Introduction of additional transfer phenomena such as a magnetic field, let appear important flow's structure modifications and temperature field distortion for large angles. It is to suspect then that long distance interactions such as in moderate emitting-absorbing semi-transparent media would severely alter natural convection inside diode cavities. As shown for emitting-reflecting surfaces, the effects of radiation in non-participating media are significantly demonstrated in diode cavities. For instance, the radiative transfer influence has been examined with the help of the radiosity technique. Baïri et al. [8] note that in a diode cavity, presence of radiation incoming from the interfaces strongly affects the natural convection and may reduce it substantially for particular angles. The diode cavity has also been exploited, due to its electromagnetic chaotic behaviour analogous to modes of an electron in a resonant tunnelling diode, to analyse the geometrical ray dynamics when the parallelogrammic cell is bounded by specularly reflecting surfaces and filled with a gradient refractive index medium in which the Fermat's principle applies [9]. Although several complex geometries have been numerically studied with various techniques such as meshless methods [10], when radiation holds in the cavity, the case of the parallelogrammic shape has not been, to our best knowledge, examined when filled with a semi-transparent absorbing-emitting medium.

The most precise techniques in radiative transfer describe the energy transfer along the photons paths, like Monte-Carlo or ray tracing methods, in simple or complex geometries [11,12]. However, due to the high number of rays to be generated in order to obtain a good accuracy, such techniques are not well suited for combined heat transfer including radiation. The discrete ordinates methods (DOM) are strongly dependent on the quadratures used to compute the angular integrals and suffer from inherent associated ray effects phenomena examined in the literature [13,14]. Nevertheless these techniques remain extremely popular due to their ease of implementation, essentially in structured rectangular grids. Mishra

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Nomenclature

Bis_n, Cis_n	Altaç angular integrated Bickley-Naylor functions
$(\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z)$	unit vectors of the x, y, z directions
G	volumic incident radiation (Wm^{-3})
H_x	length of the cavity sides along the x direction (m)
H_y	length of the cavity sides along the $\hat{y}$ direction (m)
(i, j)	internal cells numbering
$I_{ij}(\boldsymbol{\Omega})$	intensity at the (i, j) cell centre ($Wm^{-2}Sr^{-1}$)
Ki_n	Bickley-Naylor functions
N_x	cells number on the sides parallel to the x direction
N_y	cells number on the sides parallel to the y direction
$\boldsymbol{q}_{ij}^r$	radiative flux vector at the (i, j) cell centre ()
q^x	x-component of the radiative flux ()
q^y	y-component of the radiative flux ()
S	volumic radiative source (Wm^{-3})
T	temperature (K)
x, y, z	coordinate axis directions

Greek letters

Δx	characteristic cell length along the x direction (m)
Δy	characteristic cell length along the y direction (m)
ε	surface emissivity
ρ	surface reflection factor
κ	absorption coefficient (m^{-1})
σ	Stephan-Boltzmann constant (5.67 10^{-8} $Wm^{-2}K^{-4}$)
τ	optical depth
ϕ, θ	angular description of the unit vector $\boldsymbol{\Omega}$
$\boldsymbol{\Omega}$	unit vector of radiation propagation
Σ_O, Σ_N, Σ_E, Σ_S	western, norther, eastern and southern boundary surfaces of the cavity

Subscripts (superscripts)

E, N, O, S	east, north, west and south

et al. [15] proposed an improved DOM by choosing the quadrature angles and weights leading to accurate results, which partially removes the ray effects. Another problem due to false scattering produces non-physical oscillations due to negative resulting intensities when the characteristic optical depths are small [16]. Many decades ago, Crosbie et al. [17] implemented a numerical integration of the integral equations describing the radiative transfer in a rectangular two-dimensional cavity filled with a semi-transparent cold absorbing and scattering medium to avoid these ray-effect phenomena. Altaç et al. [18–20] extended the work of [17] by introducing a set of special analytical functions, and were able to produce extremely accurate values of the incident radiation and radiative flux inside a scattering absorbing medium bounded by cold surfaces. Undoubtedly, the formulation detailed by Altaç in Ref. [19], especially when the boundary surfaces are diffuse reflectors, has a strong and major advantage on other numerical techniques such as discrete ordinates methods, by transforming an angular integral representation of the incident radiation and/or radiative vector flux, by a purely space volume and surface integrals. Indeed, transformation of the angular integrals into spatial ones completely eliminates the ray effect inherent to discrete ordinates methods, and may particularly be appropriate for evaluating the radiative fields, by using Meshless techniques [10,21], in geometries of complex shapes. In the present paper we completely describe the radiative field inside a semi-transparent medium bounded by a parallelogrammic cavity in a partially analytic way, from analytical developments by keeping a hybrid formulation combining space and angular integrals as in Refs. [22,23]. Although diffusely reflecting surface radiation has been taken into account to quantify the radiative transfer influence on the natural convection in a diode cavity [8], to our best knowledge similar studies have not been investigated yet if including the semi-transparent media radiation emission-absorption effect when the boundary surfaces are diffusely reflecting. Simultaneous influence of both the absorption coefficient of the medium and the reflection on the boundary surfaces of the cavity, either diffuse or specular, can strongly alter the radiative behaviour of a semi-transparent medium compared to black surfaces enclosures [25–27] for cylindrical or spherical annuli. It is then suspected that presence of reflecting surfaces cavity may also strongly modify the internal radiative transfer inside absorbing-emitting semi-transparent media. However such studies are limited to one-dimensional devices with relatively simple analytical formulation, and are not well suited for two-dimensional cavities of any shape. In this present study, we shall develop the expressions of the radiosity temperatures on the cavity's diffusely reflecting surfaces which shall be useful to compute both the incident radiation and radiative flux fields for the determination of the temperature field at radiative equilibrium: this is the extension of [23] when taking into account the reflection on the surfaces. The numerical treatment combines a discrete derivation of the useful integrals and an iterative scheme to compute both the radiosity surface temperatures and the internal temperature field. This extension is efficient to calculate temperature and flux fields inside an absorbing and emitting medium bounded by both isothermal and adiabatic diffusely reflecting surfaces.

This paper is organized as follows: in section II we develop the exact expressions of the local radiosity temperatures on the reflecting/emitting boundary surfaces, different from the real temperatures due to the presence of diffuse reflection, so as the radiative source and flux field when using a hybrid formulation combining spatial and angular integrals. In section III we explain the angular and spatial discretisation of the useful integrals. Finally we present

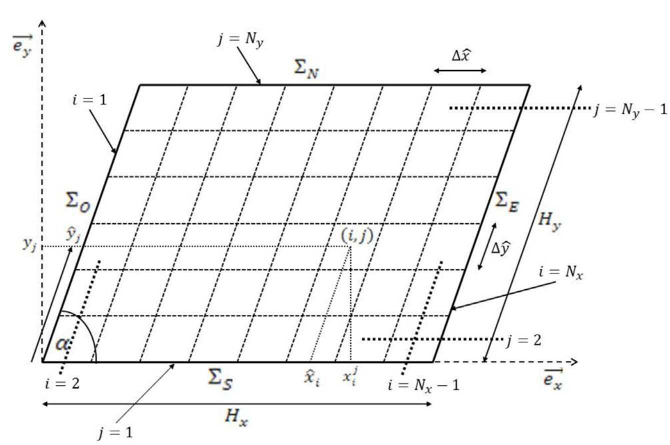

Fig. 1. Description of the parallelogrammic cavity's geometry.