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Theoretical study of electro-thermo-acoustic instability in a magneto-plasma-dynamic thruster

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ABSTRACT

The electro-thermo-acoustic instability has been hypothesized as a driver behind the near-anode plasma flow instability in the magneto-plasma-dynamic thrusters. In order to explain how this instability can be triggered, an extended Rayleigh's criterion emerging from the source term of a developed acoustic energy equation has been derived. This criterion states that the fluctuations of pressure (or velocity) and electric field in the sheath region must be in phase for the instability to be excited by the acoustics/plasma coupling. In addition, a characteristic equation deduced from a generalized wave equation has been solved to determine the growth rates and actual frequencies of unstable modes in terms of time lag between the plasma waves. By using this mathematical model, the plasma instability near the anode of Princeton benchmark thruster has been analyzed for the ratios of the squared discharge current to mass flow rate of 16, 64, and 144 $(\mathrm{kA})^2{\cdot}s/g$. As a significant result, it was shown that the calculated growth rates were positive, meaning that the electro-thermo-acoustic instabilities can arise at the all discharge current levels. This phenomenon has been justified by providing two mechanisms based on the Ohmic heating effect and anode starvation process, respectively.

1. Introduction

The self-field magneto-plasma-dynamic (MPD) thruster is one of the promising electric propulsion systems, which can be employed for mid and long space missions. Considering its high specific impulse in comparison with other electric thrusters, it also achieves high thrust density. As schematically shown in Fig. 1, the Lorentz force generated by the interaction between the electric current and the self-induced magnetic field is the driving force given to the propellant of MPD thrusters. According to the experimental investigations, the self-field MPD thrusters still have a low efficiency [1]. In order to achieve high thrust, specific impulse and consequently high efficiency, the operating parameter defined as the ratio of squared discharge current to mass flow rate $I^2/\dot{m}$ must be large. But there is a limitation in the attempt of increasing current level at the constant mass flow rate. At the point beyond the critical value of $(I^2/\dot{m})^*$, it was found that an unstable regime would occur, the so-called onset phenomenon [2]. A strongly unstable regime leads to a profound change of the dynamics of the plasma, to an apparently erratic electrical response (indicating high-amplitude voltage fluctuations) of the thruster, and intense erosion of thruster components, which can diminish the lifetime of the thruster.

Several theories have been put forward to describe the origins of the onset of unstable operation. However, there are two categories of well-supported theories which are responsible for the possible instabilities in the MPD thrusters. In the first category, the anode starvation phenomenon discussed in some experimental investigations [3–6] is often a triggering mechanism for the plasma destabilization. In this mechanism, dominant Hall effect near the anode surface causes the high axial current to be produced, leading to Lorentz pumping force which drives the charge carriers away from the anode region. Due to this decrease in the number density of charge carriers in the vicinity of anode, the total currents imposed by external source cannot be collected by the anode. Thus, as the near-anode plasma density drops and the current rises, the anode enters sheath-limited current saturation. Accordingly, attempts to conduct current greater than the sheath-limited current give rise to onset phenomenon. In order to carry increasing thruster current in the anode-starved onset condition, large electric fields accompanied with high voltage drops are created through the sheath region to supply the current density at the necessary levels. The importance of the Hall effect and sheath behavior on the anode starvation has been also studied numerically in Refs. [7–10]. Aside from the role of Hall effect and sheath phenomenon in the evolution of anode starvation, another mechanism so-called anode spotting has been also proposed to explain the starvation origin. As stated by Diamant et al. [11], the discharge through transition from a diffuse, low voltage and low noise mode at the anode, to a mode in which the current attaches in spots is required for resolving the anode starvation crisis. It has been identified that the current collection at the anode is diffuse when the

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Nomenclature

a	Speed of sound, m/s
e	Charge of an electron, 1.602×10^{-19} C
E	Electric field strength, Vm^{-1}
$\mathbf{E}$	Electric field Vector, Vm^{-1}
$\mathbf{F}_c$	Electrostatic force, N/m^3
I	Total discharge current, kA
J	Electric current density, Am^{-2}
$\mathbf{J}$	Electric current vector, Am^{-2}
k_B	Boltzmann's constant, 1.381×10^{-23} JK^{-1}
L	Half length of anode lip, m
$\dot{m}$	Propellant mass flow rate, gr/s
m	Mass of particle, kg
n	Number density of particle, m^{-3}
p	Pressure, Pa
$\dot{Q}$	Ohmic heating, W/m^3
r	Wave number, m^{-1}
t	Time, s
T	Temperature, K
u	Velocity component, m/s
$\mathbf{u}$	Velocity vector, m/s
σ	Electric conductivity, $(\Omega\cdot\text{m})^{-1}$
λ	Thermal conductivity, W/K/m
ε_0	Permittivity of free space, 8.85×10^{-12} Fm^{-1}
ρ	Density, kg/m^3
γ	Specific heats ratio
τ	Stress tensor, N/m^2
ϕ	Electric potential, V

Subscripts

e	Electrons
i	Ions
x	Axial direction

ratio of the near-anode current density to the highest current density resulting from the electrons thermal motion is slightly lower than one. As this ratio exceeds unity, the spot marks appear on the anode, increasing the plasma density by the anode evaporation caused by highly intense energy fluxes directed from the spots towards the anode surfaces. The frequent processes of spots formation and extinction lead to the fluctuations in the thruster's terminal voltage.

In the second category, the plasma instabilities have been identified as the possible drivers behind the onset phenomenon and flow destabilizations in the MPD thrusters. The theories presented in this branch consider the conditions in which the unstable modes of plasma flow can be developed in these thrusters. They normally propose the criterion expressions to predict the relevant threshold parameters for instabilities inception. Several instabilities have been studied as the possible contributors to the onset phenomenon: the drift instabilities resulting from increasing the relative velocities between the electrons and ions to a threshold value [12], the electron acoustic wave instability arising from the high heat transfer from the electrons to the neutrals at the existence of large gradients of neutrals [13], the modified two-stream instability caused by the existence of electric current flowing transverse to the electric field [14], the magneto-hydrodynamic supersonic wave instability which can be excited when the local Mach number reaches to one in the thruster [15], the lower hybrid drift micro-instabilities occurred by means of transferring momentum to the plasma ions via the current electrons through a dissipative process created by the coulomb collisions [16–18], and the three space charge, ion acoustic and basic electron drift instabilities driven by the diode instability and anomalous thermal energy of the electrons [19–21]. Di Vita et al. [22] showed that when the electron Hall parameter exceeds a critical value slightly larger than unity, collisions with the neutrals induces instability in the axisymmetric plasma sheath, which leads to the self-focusing of the current density into thin filaments. In the recent experimental work investigated by Uribarri et al. [23], the unstable plasma regime was attributed to the formation of anode spot which supplies a local region on the anode surface with the higher conductivity than the diffuse current regions. It was suggested that the anode spots result from the filamentation instability in which the current to the anode begins to fragment into many luminous plasma channels. Giannelli et al. [24] considered a simple mathematical model for the stability analysis of current-carrying plasma in the coaxial MPD thrusters. Their analytical investigation was in some extend similar to the work of Niknam et al. [25]. It was shown that the azimuthal symmetry breaking of the equilibrium plasma creates the current filamentation, leading to the spotty current pattern. Recently, by developing a simple linearized model, Ahangar [26] indicated that there exist the stationary oscillations of voltage, current and electron number densities through the sheath region in the filamentation regime. It was also shown that the high-conductivity spots lead to a decrease in the sheath voltage and an increase in the electron number density.

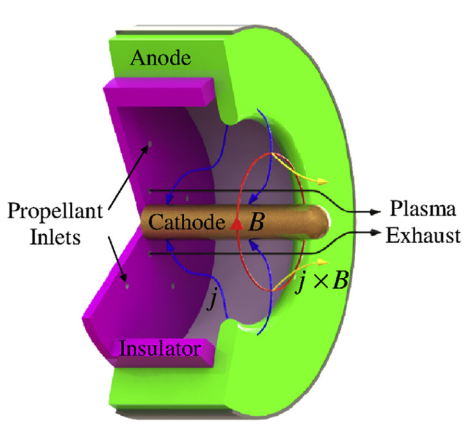

Fig. 1. Schematic view of a typical MPD thruster.

In this work, to clarify the possible origins and processes of plasma instability from the new aspects, a novel mechanism founded upon the coupling of acoustics and plasma flow will be introduced to analyze the instabilities near the anode in the Princeton benchmark thruster experimentally studied by Uribarri [27]. A notable feature of the onset phenomenon illuminated by Uribarri is that the current pattern tends to be spotty even when the operating parameter value is below its critical value. Therefore, the identification of mechanism behind the unstable plasma regime in this thruster and its correlation with the anode spotting at the almost all $I^2/\dot{m}$ levels is a worthwhile achievement which will be taken into account as the main goal of the present study. So, a criterion will be developed in the next section to show that the instability can result from the interaction of electro-thermal energy (or electrostatic force) fluctuation with the pressure (or velocity) fluctuation. Then, in section 3, a characteristic equation will be formulated for the first time to obtain the frequencies and growth rates of unstable modes in the plasma sheath region. Also, section 4 will be devoted for discussing on the results obtained by solving the characteristic equation. Finally, the conclusions of this investigation will be summarized in section 5.