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Natural convection heat transfer modeling by the cascaded thermal lattice Boltzmann method

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ABSTRACT

In this article, we present application of the Cascaded Thermal Lattice Boltzmann Method (CTLBM) in simulations of natural convection in differentially heated square cavity with adiabatic top and bottom walls. This classical benchmark problem is solved for wide range of Rayleigh numbers ($10^6 - 10^{10}$) and compared with data from the literature. For high Rayleigh numbers we present comparison of Nusselt number and wall shear stress distributions along hot wall with experimental and direct numerical simulation (DNS) data. Results for Rayleigh numbers up to 10^6 are also compared with previous results obtained by MRT-LBM simulations of Wang et al. The results are in good agreement with the existing ones obtained numerically and experimentally.

1. Introduction

Lattice Boltzmann methods have established themselves as the viable alternative among numerical methods used in CFD. LBM solves the discretized Boltzmann Transport Equation (BTE) to obtain set of distribution functions (DFs), from which macroscopic quantities (density, pressure, velocity, temperature) are then obtained. The physics solved by the LBM is controlled by the form of the collision operator and chosen equilibria for the DFs. Several types of realization of collision operators have emerged [10,22,24,53]. The simplest collision operator is the Single Relaxation Time (SRT) sometimes called BGK in the LBM community, after the authors Bhatnagar, Gross and Krook [44]. A large number of heat transfer and fluid flow problems solved by SRT LBM are reported in the literature [5,7,11]. In the SRT approach, all non-conserved moments relax to their equilibrium with the same relaxation time (due to the construction of the collision operator). SRT is based on the BGK approach [22], which could produce numerical instabilities, when the lattice resolution is insufficient [40] and also the truncation error control is limited [56,57]. In order to increase the stability and accuracy of the LBM schemes, Multiple Relaxation Times (MRT) methods were proposed [10]. In MRT schemes collisions are performed in moment space and different moments could be relaxed with different relaxation times. MRT methods performed reasonably well and showed greater stability and accuracy compared to SRT LBM. Unfortunately the MRT methods are unstable for high Re flows and have other problems mentioned in Ref. [25]. The CLBM are methods where central moments are relaxed in a “cascaded manner“ [23,30]. CLBM was successfully used for high Re fluid flow and general heat transfer problems [2,18,19,21,24,37,46].

Some authors use MRT methods for the fluid flow and e.g. finite differences to solve energy equation independently [6,45]. This approach is known as Hybrid LBM. Other approach known as Double Distribution Function (DDF) scheme was proposed by Ref. [42], here two sets of DFs are used, one for the Navier-Stokes equations and another for the energy equation. A large number of research articles appeared, which describe DDF LBM approach with SRT and MRT LBM (see e.g. Refs. [40,52] and references therein). Recently, D_3Q_{27} DDF cascaded LBM was used for steady velocity field and solute transport in porous medium [20]. Another article describe CLBM scheme for the fluid flow and SRT LBM for the energy equation [21]. The double cascaded DDF LBM scheme for thermal problems was recently presented by the authors in Ref. [19], where CLBM-CTLBM approach was derived and applied to solve forced convection, meanwhile Fei et al. published somehow similar DDF CLBM approach applied to heat transfer [46].

In the present article we solve natural convection in a differentially heated cavity by the cascaded DDF LBM. We compare results obtained from our CTLBM code with data from literature. Flow and heat transfer

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in the square cavity for wide range of Rayleigh numbers have been studied by various groups and substantial research has been carried out. Various authors used finite differences, finite volumes, finite elements and pseudo-spectral methods [8,9,15,28,32–34], and also LBM [3,13,14,16,17,36,43]. Some of the researchers have adopted Hybrid LBM and DDF LBM [14]. The SRT [3,16] and MRT LBM were used for laminar flow regime [13,17,36,43]. Dixit et al. [16] used DDF LBM for solving internal energy equation along with counter-slip boundary conditions together with mesh refinement and simulated high Rayleigh number flows (up to $Ra = 10^{10}$) with SRT. Recently, Jami et al. [27] published a paper where two MRT DDF LBM schemes have been used to solve natural convection up to $Ra = 10^8$. Allen and Reis derived moment based boundary conditions and incorporated them in the MRT LBM in order to solve natural convection in a square cavity [36]. Ren et al. [35] presented the CUDA implementation of DDF LBM scheme with a SRT collision operator to solve natural convection in a square cavity with solid obstacles. Wang et al. [17] used MRT LBM to qualitatively examine natural convection in square cavity up to $Ra = 10^6$ and also Rayleigh-Bénard convection. High Ra number flows in cavities with aspect ratio close or equal to 1 were studied mainly by LES and DNS e.g. Refs. [49–51].

2. Double distribution cascaded lattice Boltzmann method for natural convection

DDF LBM approach solves two Lattice Boltzmann Equations, one describes evolution of velocity distribution functions and other describes evolution of temperature or internal energy distribution functions. They are solved simultaneously on two separate lattices. In this article, we consider approach where the temperature is considered as a passive scalar because viscous dissipation of heat and compression work are negligible. We use D_2Q_9 lattice model for fluid flow and D_2Q_5 lattice model for temperature field. The characteristic velocities of the lattice models are depicted in Fig. 1 and defined by following sets: $\mathbf{c}_i = (c_{i,x},\ c_{i,y})$ for D_2Q_9 lattice ($i = 1, \ldots, 9$) are

$$\{(0,0), (-1,1), (-1,0), (-1,-1), (0,-1), (1,-1), (1,0), (1,1), (0,1)\},$$

and for D_2Q_5 lattice ($i = 1, \ldots, 5$) are

$$\{(0,0), (-1,0), (0,-1), (1,0), (0,1)\}.$$

The weight factors, w_i for D_2Q_9 and D_2Q_5 are $w_1 = 4/9$, $w_{3,5,7,9} = 1/9$, $w_{2,4,6,8} = 1/36$ and $w_1 = 1/3$, $w_{2,3,4,5} = 1/6$, respectively.

2.1. Cascaded LBM for the flow field

The fluid behavior at mesoscopic scale is described by fluid particles in the framework of the Boltzmann's work and their properties at certain space and time are defined by moments $m_{\alpha\beta}$ of velocity distribution functions (DFs) $f(\mathbf{x}, \xi, t)$. The evolution of such DFs obeys Boltzmann Transport Equation which reads

$$\frac{\partial f(\mathbf{x}, \xi, t)}{\partial t} + \xi \cdot \nabla f(\mathbf{x}, \xi, t) = \Omega(f, f), \tag{1}$$

where ξ is the microscopic velocity, Ω is the collision operator. The spatial and temporal derivatives in BTE (1) are discretized, the velocity distribution functions are reduced to finite given by the desired lattice model, in our case the D_2Q_9 and D_2Q_5. Then by choosing the cascaded form of collision operator and incorporating forcing term $\widetilde{F}_i$ we end up with the cascaded lattice Boltzmann equation (CLBE), which in lattice units reads

$$f_i(\mathbf{x} + \mathbf{c}_i, t + 1) = f(\mathbf{x}, t) + \mathbb{K} \cdot \mathbf{k} + \widetilde{F}_i, \tag{2}$$

where f_i is the ”velocity“ distribution function linked to the i^{th} characteristic velocity, $\mathbb{K}$ is transformation matrix, $\mathbf{k}$ is a vector of moments of f_i resulting from the cascaded collision operator. The equilibrium distribution function f_i^{eq} is defined based on the Maxwell-Boltzmann equilibrium distribution function by

$$f_i^{eq} = \rho w_i \left(1 + \frac{\mathbf{u}\cdot\mathbf{c}_i}{c_s^2} + \frac{(\mathbf{u}\cdot\mathbf{c}_i)^2}{2c_s^4} - \frac{\mathbf{u}\cdot\mathbf{u}}{2c_s^2}\right), \tag{3}$$

where ρ is density, c_s is the speed of sound and $\mathbf{u} = (u, v)$ is macroscopic velocity vector. For the lattice model D_2Q_9 we have $c_s = 1/\sqrt{3}$. The body force can be modeled by different approaches, those are compared in Ref. [1] for the problem of a natural convection. We use the forcing scheme proposed by Ref. [58]. First the velocity field is modified by known force $\mathbf{\Gamma} = (\Gamma_x,\ \Gamma_y)$ (which is defined later),

$$u = \frac{1}{2\rho}\Gamma_x \quad v = \frac{1}{2\rho}\Gamma_y \tag{4}$$

then expressions for the components to be included in cascaded collisions are

$$\begin{aligned}
F_4 &= -(\Gamma_x u + \Gamma_y v) \\
F_5 &= -(\Gamma_x u - \Gamma_y v) \\
F_6 &= \tfrac{1}{2}(\Gamma_x v + \Gamma_y u) \\
F_7 &= -\tfrac{1}{2}(u^2\Gamma_y - \Gamma_x uv) \\
F_8 &= -\tfrac{1}{2}(v^2\Gamma_x - \Gamma_y uv) \\
F_9 &= -(\Gamma_x uv^2 + \Gamma_y u^2 v)
\end{aligned} \tag{5}$$

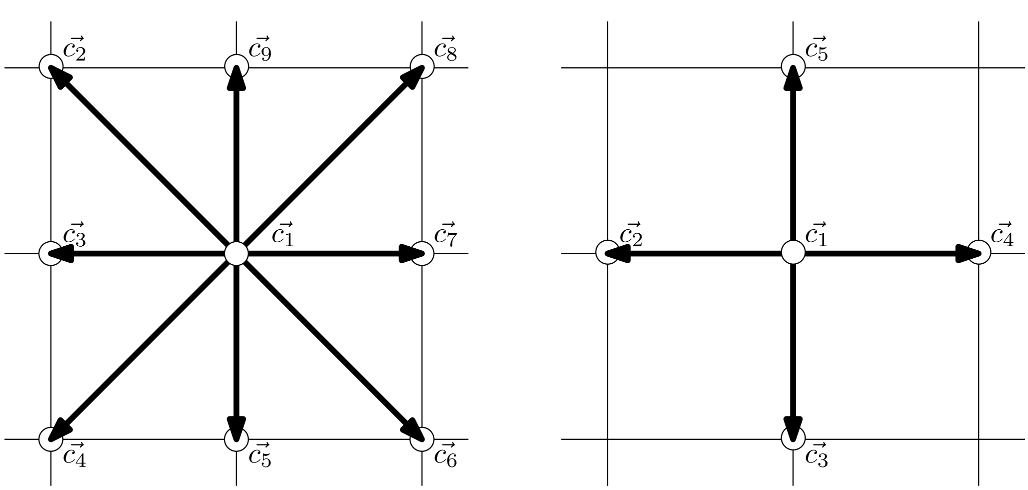

Fig. 1. Characteristic velocities (links) in 2D for D_2Q_9 (left) and D_2Q_5 (right) lattice models.