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Revisited analysis of gas convection and heat transfer in micro channels: Influence of viscous stress power at wall on Nusselt number

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ABSTRACT

This paper deals with the modeling of weakly rarefied and dilute gas flows in micro channels by the continuum approach, valid for Knudsen numbers smaller than about 0.1. It particularly focuses on the modeling of the associated heat transfer. The models proposed in the literature for the forced convection of gas flows in long micro channels between two infinite plates are more specifically discussed. The complete model for such flows is reminded after a compilation and a brief description of their possible applications in industries. The compatibility of the pressure work and viscous dissipation in the energy equation with the power of the viscous stress at the walls is discussed in detail. A dimensional analysis is proposed in the context of long micro channels. Analytical solutions for the velocity and temperature fields and for the Nusselt number are provided in the case of compressible micro-flows in isothermally heated flat plate channels, with pressure work and viscous dissipation included. The choice of an appropriate Nusselt number, including the power of the viscous stress at the wall, is particularly discussed. It is shown analytically and numerically, by solving the complete model for an isothermal wall micro-channel, that the Nusselt number tends to zero when the hydraulic diameter decreases, that is when the Reynolds number decreases and the Knudsen number increases. This could theoretically explain the very small values of the Nusselt number obtained in the experiments by Demsis et al. (2009, 2010).

1. Introduction

Due to the increasing development of Micro Electro Mechanical Systems (MEMS), the study of liquid or gas flows and heat transfer in ducts, heated or not, whose hydraulic diameter, D_h, is of the order of a few microns (say 1 to 100 μm), has given rise to a considerable amount of works over the past twenty years. A recent review by Kandlikar et al. [1] is dedicated to them. It is shown that monophasic liquid flows in micro channels have a behavior similar to that observed at the macroscopic scale and the classical continuum mechanics model can be used (Navier-Stokes equations with no slip boundary conditions).

However, for gas flows at the microscopic scale, specific phenomena are observed and require appropriate models [2,3]. A slightly rarefied flow regime close to the wall, generated by the interaction between the gas molecules and the wall atoms, must be taken into account at the microscopic scale whereas it is negligible at the macroscopic scale or for liquid micro-flows. More specifically, a Knudsen layer whose thickness is of the order of the mean free path of the gas molecules, λ, is formed closed to the wall. In this layer, the velocity magnitudes of the gas molecules considered individually are different at a fixed distance from the wall, due to their interactions with the wall. In other words, in this layer, the gas is in a state of local thermodynamic non-equilibrium which results in non-linear mean velocity profiles and relations between stress and strain rates. From the continuum mechanics point of view, at the micro channel scale, when the Knudsen number is such that $0.001 < Kn = \lambda/D_h < 0.1$, these phenomena translate into a slip velocity and a temperature jump at the wall and, possibly, a gas flow driven by the tangential temperature gradient along the wall called "thermal creep" [4]. The consequences of these phenomena on the macroscopic quantities such as the mass flow rate, the friction factor, the bulk temperature and the wall heat flux can be significant [5] and must be taken into account in the modeling of the convective heat transfer in MEMS with gas flows. Indeed they may have antagonistic effects on the heat transfer.

Gas micro-flows, possibly with heat transfer, can be found in:

- micro heat exchangers for the cooling of electronic components or in chemistry [6,7],
- micro pumps and turbines, including the thermal transpiration-driven Knudsen pumps for vacuum pumping applications [8–10],
- micro-systems for the species separation in gas mixtures such as the method of gas separation by membranes [11],

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•micro gas analyzers such as micro mass spectrometers and micro-chromatographs [12,13],
•supersonic gas flows in micro-nozzles to control the nano-satellite attitude or the boundary layers in aerodynamics [14–18],
•artificial lungs [19,20],
•pressure, flow rate and temperature micro-sensors in gas flows [21,22], etc.

This paper investigates the theoretical models available to simulate and analyze the slightly rarefied gas micro-flows with heat transfer, when $Kn \le 0.1$. We focus on the modeling of forced convection of pure diluted gases in micro channels by a continuous approach based on the Navier-Stokes equations and first order slip and temperature jump boundary conditions. It appears that simplified models are often used in the literature for this flow type, but without relevant justification and with recurrent errors propagating from one paper to the other, particularly concerning the heat transfer analysis and the energy equation. Our purpose is to provide a consistent model for gas micro-flows and heat transfer and to compare with the vanishing values of the Nusselt number obtained in experiments [23,24].

In that aim, the characteristic length scales of the continuum description and the way of modeling the Knudsen layer are reminded in §2. The values of the slip and temperature jump coefficients are particularly discussed. The complete model for forced convection in heated micro channels is established and discussed in §3. A dimensional analysis is developed and the analytical solution of the temperature field given by a simplified asymptotic model for compressible gas convection in an isothermal micro-channel is established in §4. This solution is compared with the numerical solution of the full model obtained from finite volume simulations in §5. Furthermore, from the numerical simulations, the heat flux balances for slip and no slip flows and incompressible and compressible flows are analysed in details. The numerical method to solve the full model is presented in §5.1 and the analytical and numerical solutions are compared in §5.2. The heat flux balances and the very small values of the Nusselt number obtained in the experiments by Demsis et al. [23,24] are explained in §5.3.

2. Length scales of the continuum description and Knudsen layer modeling

The mean free path, λ, is the average distance traveled by the molecules between two successive collisions. It is the main scale to evaluate the rarefaction rate in a gas flow and the validity domain of the continuum description. In this paper, the most standard definition used for ideal gases is retained [2,5,26–28]:

$$\lambda = \frac{\mu}{p}\sqrt{\frac{\pi r T}{2}} = \frac{\mu}{\rho}\sqrt{\frac{\pi}{2rT}} = \mu\sqrt{\frac{\pi}{2p\rho}} \tag{1}$$

where r is the specific gas constant.

A scale analysis of the breakdown of the continuum description of gas flows was presented by Bird [25] and recalled by Gad-El-Hak [2,29], Colin [30] and Zhang et al. [31]. For dilute gases, the limit on the range of validity of the continuum equations is first due to the local thermodynamic non-equilibrium and the presence of very steep gradients in the flow fields (on characteristic lengths, L, of the same order as the mean free path, λ), rather than due to statistical fluctuations of the macroscopic variables. It is generally admitted that the thermodynamic equilibrium is satisfied and the Navier-Stokes equations are valid everywhere for $Kn = \lambda/D_h < 0.001$ to 0.01, and in the flow core only for $0.01 < Kn < 0.1$. In the latter case, in the thin layer close to the wall (the Knudsen layer), the continuum equations are not valid because the gas molecules only "see" a half-space where the nature of the shocks with the wall is different from the inter-molecular shocks: a local thermodynamic non-equilibrium is present in this zone.

An accurate description of the thin Knudsen layer whose thickness is between about λ and 3λ [5,31] is crucial for microfluidics applications. Momentum and energy are indeed transferred between the gas and the wall through this layer. An ill description has thus significant consequences on the evaluations of the mass flow rate and friction factor, or on the maximum bulk temperature and wall heat flux. For $0.001 < Kn < 0.1$, a continuous approach coupled with a modeling of the flow and heat transfer in the Knudsen layer is generally considered. The most used model consists in solving the Navier-Stokes equations with slip boundary conditions and semi-empirical coefficients, such as the "accommodation coefficient" or the "slip length", to model the gas/wall interaction. A similar model is used for heat transfer: the energy equation is solved with a temperature jump boundary condition to mimic the thermal resistance of the Knudsen layer. The choice of the slip and temperature jump coefficients is detailed below because inadequate or unconsistent values of these coefficients are regularly used in the literature.

2.1. Slip and thermal creep boundary conditions

The first-order slip boundary condition was first introduced by Navier in 1823 [36], then independently by Maxwell in 1879 [37]. The simplified form on an impermeable wall writes:

$$u_g - u_w = L_s \left.\frac{\partial u}{\partial \vec{n}}\right|_g \tag{2}$$

$$v_g = 0 \tag{3}$$

where u and v are the velocity components of the gas, tangential and normal to the wall respectively, u_g is the slip velocity of the gas on the wall, u_w is the velocity of the wall ($u_w = 0$ in this study), L_s is the slip length and $\vec{n}$ is the direction normal to the wall directed toward the gas with $\partial u/\partial\vec{n} = \nabla u.\ \vec{n}$. Here and in the following, the subscript "g" is used to denote quantities on the gas side of the wall (slip-related quantities associated with the gas molecules in contact with the wall) and the subscript "w" is used to denote the quantities on the solid side of the wall.

The slip boundary condition (2) and the comparison of the "true" and modeled velocity profiles of the gas in the Knudsen layer are illustrated in the left part of Fig. 1. The "true" velocity profile (in red) is non-linear close to the wall and presents a slip speed at the wall denoted $u_{g,true}$. The Navier and Maxwell model consists in approximating this velocity profile in the Knudsen layer by the blue profile whose slip velocity, u_g, is greater than the actual slip velocity, $u_{g,true}$, by considering that the difference $u_g - u_w$ is proportional to the normal velocity gradient in the Knudsen layer, $\partial u/\partial\vec{n}|_g$. The slip length, L_s, in this model is a semi empirical parameter, proportional to the mean free path, λ, and depends on the gas and wall nature, on the wall roughness and, more generally, on the type of gas/wall interaction (diffuse, specular or mixed specular and diffuse scattering of the gas molecules at the wall). L_s must be evaluated so that u_g provides a good

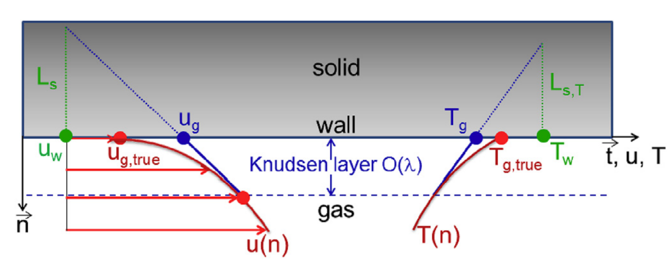

Fig. 1. Schematic representation of the true (in red) and approximated (in blue) velocity and temperature profiles in the Knudsen layer and (in green) of the slip length, L_s, and temperature jump length, $L_{s,T}$. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)