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Pair-particle trajectories in a shear flow of a Bingham fluid

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A B S T R A C T

We study numerically the pair trajectories of rigid circular particles in a two dimensional inertialess simple shear flow of a (Binghamian) yield stress fluid. We use a Lagrange multiplier based fictitious domain method, following Glowinski et al. [26, 28, 29], for solving the problem. Contacts between the particles at a finite interparticle distance interpreted as a roughness are taken into account with the da Cunha and Hinch [12] model. Another model, introduced by Glowinski et al. [27], is shown to provide similar results in the limit of infinite contact stiffness. Due to the nonlinear behavior of the suspending fluid, it is found that the trajectories of the particles depend on the shear rate, the relevant dimensionless parameter being here the Bingham number, which compares plastic forces to viscous forces. In absence of interparticle contacts, fore-aft symmetry is observed in all cases; however, the particles are found to come closer to each other as the Bingham number is increased: the plastic behavior of the suspending fluid decreases the range of hydrodynamic interactions. As contacts are introduced, fore-aft asymmetry is observed. Plastic effects are found to enhance surface roughness effects: contacts between particles occur for smaller surface roughness at large *Bn*. Moreover, the magnitude of the asymmetry is increased as the Bingham number is increased. These observations may explain why the microstructure of suspensions of particles in a yield stress fluid is shear-rate-dependent [45] leading to a complex nonlinear macroscopic behavior.

1. Introduction

Suspensions of non-Brownian and noncolloidal hard spheres are relevant model systems to help understanding the behavior of complex polydisperse suspensions found in the industry (fresh concrete...) and in geophysics (debris-flows...) [1,11,20,25,36,40,55].

Substantial progress in the understanding of the behavior of such materials can thus be made by studying the impact of adding noncolloidal particles to a yield stress fluid of known properties [8,13,39,45,57]. From a more fundamental point of view, these systems, viewed as rigid inclusions in a nonlinear material, may provide crucial tests for micromechanical approaches developed to describe the behavior of suspensions and composite materials.

A key element in the understanding and modeling of the macroscopic behavior of suspensions is the hydrodynamic interaction between particles and its impact on the suspensions' microstructure [1,7]. Many information can readily be obtained by studying a seemingly simple problem involving interactions between a single pair of particles, that is, the study of the relative trajectories of two particles in a simple shear flow.

The case of particles in an inertialess Newtonian fluid has been studied in much detail [30,43]. For perfectly smooth particles, two families of trajectories exist. At close interparticle distance, particles are found to orbit around each other; otherwise, they cross each other by following a symmetric trajectory in the frame centered on their neighbor, which comes from the linearity of the Stokes equation and the symmetries of the problem. Such symmetry is not observed experimentally for non-perfectly smooth particles [49]. This has been attributed to the finite roughness of the particles, which introduces a cut-off in the hydrodynamic interaction between the particles, which now makes the problem nonlinear. Consequently, the fore-aft symmetry is broken. It is finally found that the modeling of the trajectories based on the experimentally measured roughness allows to describe accurately experimental results [6]. At the scale of suspensions, this induces asymmetric pair distributions functions (pdfs) and nonzero normal stress differences [43]. Again, when the experimentally measured roughness is taken into account to model the hydrodynamic interactions between pairs of particles, very good agreement is found between the experimentally measured and numerically computed pdfs [5].

The study of particles in Newtonian fluids has been extended to inertial flows [30,31,38]. In this last case, the fore-aft symmetry is also found to be broken, due to the nonlinear convective acceleration; the magnitude of the asymmetry then depends on the Reynolds number. This has been found to be accompanied with the vanishing of closed particle trajectories [38], and to the emergence of new forms of trajectories. All these features are predicted to impact the rheological behavior of inertial suspensions, and to lead to observable Reynolds number dependent rheological properties [31].

Particles in non-Newtonian fluids have been much less studied, although the case of particles suspended in viscoelastic fluids have received a lot of attention recently, in particular because of the tendency of particles to form chains under flow [15]. Since yield stress fluids are used in the industry to stabilize the particles against gravity, the sedimentation problem has been the subject of most studies on particles suspended in yield stress fluids [4,22,46,53], with a focus on a single particle and on the onset of flow. The rheological properties of model

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suspensions have also been studied recently experimentally [13,39,45] and theoretically [3,8]. The method used by Chateau et al. [8], valid for any generalized Newtonian behavior, consists, first, in assuming that the overall properties of the suspension can be accurately estimated from an average estimate $\bar{\dot{\gamma}}_{\text{local}}$ of the local shear rate over the suspending fluid domain, second, in using one of the material properties determined experimentally to estimate the value of $\bar{\dot{\gamma}}_{\text{local}}$, and finally, in using this last value to predict the other properties. Estimates of the suspension apparent viscosity have then been obtained by linearizing the suspending fluid behavior at each prescribed macroscopic shear rate and computing its effective viscosity at the average local shear rate $\bar{\dot{\gamma}}_{\text{local}}$. A major result is then the interdependence of the evolutions of all rheological properties with the particle volume fraction: it seems that all rheological properties can be predicted as soon as one of them is known. This has first been validated experimentally on isotropic suspensions by Mahaut et al. [39]. However, some discrepancy between the rheological properties evolutions observed at low and high shear rates was reported by Ovarlez et al. [45] for sheared suspensions. This discrepancy was attributed to the emergence of a shear-rate-dependent microstructure for the suspensions, as observed experimentally; it was indeed pointed out that the interdependence between rheological properties at low and high shear rate is obtained theoretically only under the assumption that the microstructure is shear-rate-independent. The observed shear-rate-dependent microstructure can have a large impact on the rheological behavior; e.g., it was found that the stress at flow onset is strongly dependent on shear history.

To better understand the origin of a shear-rate-dependent microstructure in yield stress fluids, we study numerically the hydrodynamic interaction of two particles in an inertialess simple shear flow of a (Binghamian) yield stress fluid in two dimensions (2D). Pair trajectories are computed by solving a Lagrange multiplier based fictitious domain method, following Glowinski et al. [26, 28, 29]. Contacts between the particles at a finite interparticle distance interpreted as a roughness are taken into account with the da Cunha and Hinch [12] model. Another model, introduced by Glowinski et al. [27], is shown to provide similar results in the limit of infinite contact stiffness.

In Section 2, we first present precisely the studied problem. The numerical method used to solve the problem is then described in detail in Section 3. The results are finally shown and discussed in Section 4 before conclusions.

2. Mathematical model

We study the interaction of two neutrally buoyant and equal-sized circular particles in $\mathbb{R}^2$ suspended in an incompressible Bingham fluid undergoing a simple shear flow of shear rate $\dot{\gamma}$. In the simulations, we approximate this problem by solving the equations in a box with boundary conditions chosen to ensure that the average shear rate is $\dot{\gamma}$. It is then expected that, for large boxes, this might provide a good approximation of the solution for an infinite domain.

A sketch of the two-dimensional problem that we solve is shown in Fig. 1: two particles of radius a, denoted by $P_1(t)$ and $P_2(t)$, are placed symmetrically in a rectangular box, Ω, of size $L \times W$. The collective particle region at a certain time t is denoted by $P(t) = P_1(t) \cup P_2(t)$. External boundaries are denoted by $\Gamma = \partial\Omega = \bigcup_{i=1}^{4} \Gamma_i$ and the boundary of the ith particle by $\partial P_i(t)$. The origin of the coordinate system is located at the center of the domain; x is the flow direction and y is the velocity gradient direction. On the upper and lower boundaries, equal and opposite velocities, $\pm U_W = \pm\dot{\gamma}W/2$, are prescribed. Periodic boundary conditions are prescribed on the left and right boundaries. We refer to the initial perpendicular distance between the line of motion and the x-axis as the *initial offset* and denote it by $y_{-\infty}$ (e.g., it corresponds to the far upstream y-coordinate of $P_1(t)$, as $x \to -\infty$). Changing the value of the parameter $y_{-\infty}$ allows to tune the strength of hydrodynamic interaction between the two particles. We neglect inertia and body forces for both the fluid and the particles.

Hence, the motion of the fluid is governed by the Stokes set of equations

$$\nabla \cdot \boldsymbol{\sigma} = 0 \quad \text{in } \Omega \backslash P(t), \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0 \quad \text{in } \Omega \backslash P(t), \tag{2}$$

where $\boldsymbol{u}$ is the fluid velocity and $\boldsymbol{\sigma}$ is the total stress tensor. Eqs. (1) and (2) are the balance equations of momentum and mass, respectively. For a viscoplastic Bingham fluid, $\boldsymbol{\sigma}$ is split into spherical and deviatoric parts:

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \boldsymbol{\tau}, \tag{3}$$

where p is the pressure, $\boldsymbol{I}$ is the second-order identity tensor, and $\boldsymbol{\tau}$ is the deviatoric stress tensor.

The constitutive law for the Bingham fluid reads

$$\begin{aligned} &\boldsymbol{\tau}(\boldsymbol{u}) = 2\eta\boldsymbol{D}(\boldsymbol{u}) + \tau_0 \frac{\boldsymbol{D}(\boldsymbol{u})}{|\boldsymbol{D}(\boldsymbol{u})|} && \text{if } |\boldsymbol{D}(\boldsymbol{u})| \neq 0, \\ &|\boldsymbol{\tau}(\boldsymbol{u})| \le \tau_0 && \text{if } |\boldsymbol{D}(\boldsymbol{u})| = 0, \end{aligned} \tag{4}$$

or equivalently

$$\boldsymbol{D}(\boldsymbol{u}) = \begin{cases} \left(1 - \dfrac{\tau_0}{|\boldsymbol{\tau}(\boldsymbol{u})|}\right)\dfrac{\boldsymbol{\tau}(\boldsymbol{u})}{2\eta} & \text{if } |\boldsymbol{\tau}(\boldsymbol{u})| > \tau_0, \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

where τ_0 is the yield stress, η is the plastic viscosity, $\boldsymbol{D}(\boldsymbol{u}) = (\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T)/2$ is the rate of deformation tensor, and, for any tensor $\boldsymbol{\zeta} = (\zeta_{ij})$, the notation $|\zeta|$ represents the following matrix norm

$$|\zeta|^2 = \frac{1}{2}\zeta : \zeta = \frac{1}{2}\sum_{i,j}\zeta_{ij}^2.$$

Notice that, for $\tau_0 = 0$, we recover the constitutive law for Newtonian viscous fluids.

The equations that govern the motion of the ith particle are the following inertialess Newton-Euler equations

$$0 = \boldsymbol{F}_i^h + \boldsymbol{F}_i^p, \tag{6}$$

$$0 = \boldsymbol{T}_i^h + \boldsymbol{T}_i^p, \tag{7}$$

where $\boldsymbol{F}_i^p$ and $\boldsymbol{T}_i^p$ are respectively the resultant and the moment of the contact forces acting on the ith particle due to the other particle coming close to it. In the whole paper, we assume that collision between particles give rise only to normal forces, which yields that the contact torque $\boldsymbol{T}_i^p$ is zero. The detail of the contact model is presented in Section 3.2. $\boldsymbol{F}_i^h$ and $\boldsymbol{T}_i^h$ denote respectively the resultant and the moment of the hydrodynamic forces acting on the ith particle which are calculated by

$$\boldsymbol{F}_i^h = \int_{\partial P_i(t)} \boldsymbol{\sigma} \cdot \boldsymbol{n} \, \mathrm{d}s, \tag{8}$$

$$\boldsymbol{T}_i^h = \int_{\partial P_i(t)} (\boldsymbol{x} - \boldsymbol{X}_i) \times (\boldsymbol{\sigma} \cdot \boldsymbol{n}) \, \mathrm{d}s, \tag{9}$$

where $\boldsymbol{X}_i$ is the position of the center of the ith particle and $\boldsymbol{n}$ is the unit normal vector to $\partial P_i(t)$ pointing out of the particle. All the moments are computed about the center of the particles. The position $\boldsymbol{X}_i$ of the ith-particle and its angular rotation $\boldsymbol{\Theta}_i$ are obtained by integration of the kinematic equations

$$\frac{\mathrm{d}\boldsymbol{X}_i}{\mathrm{d}t} = \boldsymbol{U}_i, \quad \boldsymbol{X}_i(t=0) = \boldsymbol{X}_{i,0}, \tag{10}$$

$$\frac{\mathrm{d}\boldsymbol{\Theta}_i}{\mathrm{d}t} = \boldsymbol{\omega}_i, \quad \boldsymbol{\Theta}_i(t=0) = \boldsymbol{\Theta}_{i,0}. \tag{11}$$

where $\boldsymbol{U}_i$ and $\boldsymbol{\omega}_i$ are respectively the translational velocity and the angular velocity of the ith particle.

For circular particles, Eq. (11) is completely decoupled from the other equations and may be ignored. Finally, the following boundary conditions are needed to close the system