Automatica 97 (2018) 392–403

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Towards efficient maximum likelihood estimation of LPV-SS models☆

Pepijn Bastiaan Cox [a,*], Roland Tóth [a], Mihály Petreczky [b]

[a] Control Systems Group, Department of Electrical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
[b] Univ. Lille, CNRS, Centrale Lille, UMR 9189 - CRIStAL - Centre de Recherche en Informatique Signal et Automatique de Lille, F-59000 Lille, France

ARTICLE INFO

Article history:
Received 29 November 2016
Received in revised form 28 May 2018
Accepted 5 August 2018
Available online 14 September 2018

Keywords:
System identification
Linear parameter-varying systems
State-space representations
Realization theory
Maximum likelihood estimation

ABSTRACT

How to efficiently identify multiple-input multiple-output (MIMO) linear parameter-varying (LPV) discrete-time state-space (SS) models with affine dependence on the scheduling variable still remains an open question, as identification methods proposed in the literature suffer heavily from the curse of dimensionality and/or depend on over-restrictive approximations of the measured signal behaviors. However, obtaining an SS model of the targeted system is crucial for many LPV control synthesis methods, as these synthesis tools are almost exclusively formulated for the aforementioned representation of the system dynamics. Therefore, in this paper, we tackle the problem by combining state-of-the-art LPV input–output (IO) identification methods with an LPV-IO to LPV-SS realization scheme and a maximum likelihood refinement step. The resulting modular LPV-SS identification approach achieves statical efficiency with a relatively low computational load. The method contains the following three steps: (1) estimation of the Markov coefficient sequence of the underlying system using correlation analysis or Bayesian impulse response estimation, then (2) LPV-SS realization of the estimated coefficients by using a basis reduced Ho–Kalman method, and (3) refinement of the LPV-SS model estimate from a maximum-likelihood point of view by a gradient-based or an expectation–maximization optimization methodology. The effectiveness of the full identification scheme is demonstrated by a Monte Carlo study where our proposed method is compared to existing schemes for identifying a MIMO LPV system.

1. Introduction

The *linear parameter-varying* (LPV) modeling paradigm offers an attractive model class to capture nonlinear and/or time-varying systems with a parsimonious parametrization. The LPV model class preserves the linear signal relation between the inputs and outputs of the system, however, these linear relations are functions of a measurable, time-varying signal, the scheduling variable, denoted as p. This scheduling signal can be any combination of inputs, measurable process states, outputs, or measurable exogenous variables and, in addition, these signals can be filtered by any arbitrary functional relation. Hence, the LPV modeling paradigm can represent both non-stationary and nonlinear behavior of a wide variety of physical or chemical processes, e.g., see Bachnas, Tóth, Mesbah, and Ludlage (2014), Groot Wassink, Van de Wal, Scherer, and Bosgra (2005), van Wingerden, Houtzager, Felici, and Verhaegen (2009) and Veenman, Scherer, and Köroğlu (2009).

The majority of LPV control synthesis methods are based upon the assumption that an LPV *state-space* (SS) model of the system is available, especially with static and affine dependence of the involved matrix coefficients on the scheduling variable p, e.g., Mohammadpour and Scherer (2012). Hence, efficient identification of LPV-SS models in terms of computational load, statistical, and performance properties has intensively been researched. Conceptually, LPV identification can be performed as: (i) the interpolation of local LTI models estimated from multiple experiments around fixed operating points, i.e., with constant p, often referred to as the *local identification* setting; or (ii) a direct model estimation problem, i.e., the *global identification* setting, which requires the experimental data with a varying p which is informative to uniquely identify the considered model parameters. Accordingly, global identification approaches include scheduling dynamics, see Bachnas et al. (2014) for a detailed comparison between the two settings. In this paper, we will focus on the global setting and the identification of discrete-time models.

In the global setting, an attractive identification approach is the minimization of the ℓ_2-loss in terms of the prediction-error associated with the model. Approaches aiming at this objective are

☆ This paper has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 714663). The material in this paper was partially presented at the 2016 American Control Conference, July 6–8, 2016, Boston, MA, USA and the 1st IFAC Workshop on Linear Parameter-Varying Systems, October 7–9, 2015, Grenoble, France. This paper was recommended for publication in revised form by Associate Editor Juan I. Yuz under the direction of Editor Torsten Söderström.

* Corresponding author.
E-mail addresses: p.b.cox@tue.nl (P.B. Cox), r.toth@tue.nl (R. Tóth), mihaly.petreczky@ec-lille.fr (M. Petreczky).

https://doi.org/10.1016/j.automatica.2018.08.021
0005-1098/

often called *prediction-error methods* (PEM). Early approaches in the PEM setting were proposed under the unrealistic assumption of full state measurements (Nemani, Ravikanth, & Bamieh, 1995; Rizvi, Mohammadpour, Tóth, & Meskin, 2015). To overcome this assumption and to directly minimize the ℓ_2 loss, *gradient-based* (GB) methodologies have been introduced, e.g., see Lee and Poolla (1997), Verdult, Bergboer, and Verhaegen (2003) and Wills and Ninness (2008, 2011). Recently, an *expectation–maximization* (EM) algorithm has been developed for LPV-SS models (Wills & Ninness, 2011), extending the set of GB methods. The EM method is more robust to an inaccurate initial estimate compared to the GB PEM; however, its convergence rate is much slower near the optimum (Watson & Engle, 1983). Due to the nonlinear optimization associated with the EM and GB methods, their convergence to the *maximum-likelihood* (ML) estimate depends heavily on a proper initial seeding. Besides prediction-error identification methods, LPV grey-box (Angelis, 2001; Gáspár, Szabó, & Bokor, 2007) and LPV *set-membership* (SM) (Bianchi & Sánchez-Peña, 2009; Ceronea, Piga, & Regruto, 2013; Novara, 2011) identification approaches have been developed. Grey-box schemes require detailed knowledge of the dynamical structure of the system with only a few unknown parameters, which are often estimated by a Kalman like filtering strategy. The SM methods characterize noise and disturbances in a deterministic bounded-error compared to the stochastic description in PEM. In general, SM approaches have a significantly higher computational load compared to direct PEM and rely on convex outer-approximations. Hence, in order to achieve stochastically interpretable and computationally attractive identification of LPV-SS models, it is favorable to apply GB and EM based PEM. However, these methods require a proper initial estimate close to the global optimum (ML estimate) in order to exploit their advantageous properties (Problem 1).

To achieve initialization of direct PEM, alternative methods can be introduced that rely on realization theory by sacrificing ML properties for an estimation problem solvable via convex optimization. These methods boil down to: first identifying an LPV-IO model, with well-established methods available in the literature (e.g., see Laurain, Gilson, Tóth, & Garnier, 2010; Lopes dos Santos, Azevedo-Perdicoúlis, Novara, Ramos, & Rivera, 2011; Mohammadpour & Scherer, 2012); and, secondly, to execute an exact realization of the identified LPV-IO form to an LPV-SS model. However, such an exact realization will, in general, result in relations with rational, dynamic dependence on the scheduling variable or lead to a non-minimal state realization if the static, affine dependence is enforced to be preserved (Tóth, 2010). Moreover, such exact algebraic realization methods have a high computational cost. Recently introduced LPV realization theory based schemes, so-called *subspace identification* (SID) methods, aim to avoid the aforementioned problem by achieving data-driven state-space realization. SID schemes can apply *direct* LPV Ho–Kalman like realization (Tóth, Abbas, & Werner, 2012) to obtain the SS matrices from specific LPV-IO models that are identified by a least-squares method; or have an intermediate *projection* step, i.e., (1) identify an IO structure using convex optimization, (2) find a projection to estimate the unknown state-sequence via matrix decomposition methods, then (3) estimate the SS matrices in a least-squares fashion, e.g., see Felici, van Wingerden, and Verhaegen (2007), Larimore (2013), Lopes dos Santos, Ramos, and de Carvalho (2007) and van Wingerden and Verhaegen (2009). However, to attain a convex problem, the latter class of SID methods usually depend on over-restrictive approximations of the signal behaviors and/or the number of observed variables grows exponentially. As a consequence, the estimation problem still has a high computational demand, making it inapplicable for real-world systems (Problem 2). The aforementioned realization based schemes provide an LPV-SS model estimate which is not minimized w.r.t. any criterion and, therefore, it is not "optimal" in an ML sense. Hence, to solve Problem 1 and 2, i.e., to have efficient initialization of direct PEM methods, we require novel computationally attractive SS identification methods capable of providing estimates that are sufficiently close to the global PEM optimum.

Based on Problems 1 and 2, we can conclude that computationally and stochastically efficient identification of LPV-SS models on real-world sized problems remains still an open question. Hence, the goal of this paper is to provide a maximum likelihood identification scheme for LPV-SS models in the global, open-loop identification setting, which can provide an integrated solution for both problems. Specifically, to solve Problem 2, we propose to identify surrogate LPV *finite impulse response* (FIR) models via a novel computationally efficient *correlation analysis* (CRA) method or via an empirical MIMO Bayesian estimation technique. Then, realization of these models is accomplished via a novel basis reduced LPV Ho–Kalman scheme, which grows linearly in complexity compared to previous methods with exponential growth, which were introduced originally in Cox and Tóth (2016) and Cox, Tóth, and Petreczky (2015). Next, Problem 1 is solved by integrating the proposed pre-estimation methods into the GB and EM schemes to obtain an ML estimate. In addition, to improve the numerical properties of the GB method, we extend the enhanced Gauss–Newton method (Wills & Ninness, 2008) to the LPV setting. Combining these methods results in a novel three-step approach with a modular structure, achieving both favorable computational properties and enabling ML estimation.

This paper is organized as follows: first, LPV-SS models with general noise structure are analyzed and compared with models relaying on an innovation structure to highlight modeling limitations of the latter form considered in many LPV SID methods. Then, the considered LPV-SS identification problem is introduced (Section 2). Next, we present our modular identification method, defined in three steps: (1) estimate the FIR model of the underlying system using CRA or MIMO Bayesian estimation (Section 3), then (2) compute an LPV-SS realization based on the estimated coefficients by using a Ho–Kalman like method (Section 4), and (3) to have an ML estimate, refine the LPV-SS model by GB and/or EM optimization (Section 5). The contribution of this paper is to provide a detailed overview of the methods applied and to demonstrate that LPV identification of moderate sized models is possible with the proposed scheme. The efficiency of the combined approach is demonstrated by a Monte Carlo study and it is compared to existing LPV-SS identification schemes (Bianchi & Sánchez-Peña, 2009; Lopes dos Santos, Ramos, & Martins de Carvalho, 2008; van Wingerden & Verhaegen, 2009) (Section 6).

2. The LPV identification problem

2.1. Technical preliminaries

We define a *random variable* $\mathbf{f}$ as a measurable function $\mathbf{f} : \Omega \to \mathbb{R}^n$, which induces a probability measure $\mathbf{P}$ on $(\mathbb{R}^n, \mathscr{B}(\mathbb{R}^n))$ with an associated Borel measurable space $\mathscr{B}(\mathbb{R}^n)$ (Bogachev, 2007). As such, a realization $\nu \in \Omega$ of $\mathbf{P}$, denoted $\nu \sim \mathbf{P}$, defines a realization f of $\mathbf{f}$, i.e., $f := \mathbf{f}(\nu)$. A *stochastic process* $\mathbf{x}$ is a collection of random variables $\mathbf{x}_t : \Omega \to \mathbb{R}^n$ indexed by the set $t \in \mathbb{Z}$ (discrete time), given as $\mathbf{x} = \{\mathbf{x}_t : t \in \mathbb{Z}\}$. A realization $\nu_t \in \Omega$ of the stochastic process defines a signal trajectory $x := \{\mathbf{x}_t(\nu_t) : t \in \mathbb{Z}\}$. We call a stochastic process $\mathbf{x}$ *stationary* if the probability distribution of $\mathbf{x}_t$ and joint probability distribution of $(\mathbf{x}_t, \ldots, \mathbf{x}_{t+k})$ for any $k \in \mathbb{N}_+$ are independent of the time-index t. In addition, a stationary process consisting of uncorrelated random variables with zero mean and finite variance is called a *white noise process*. The ring of all real meromorphic functions with finite dimensional domain is denoted by $\mathscr{R}$ and the operator $\diamond : (\mathscr{R}, \mathbb{P}^{\mathbb{Z}}) \to \mathbb{R}^{\mathbb{Z}}$ denotes $(h \diamond p)_t = h(p_{t+\tau_1}, \ldots, p_t, \ldots, p_{t-\tau_2})$ with $\tau_1, \tau_2 \in \mathbb{N}_0$. The time-shift operator is denoted by q, i.e., $qx(t) = x(t+1)$, and the set $\{s, s+1, \ldots, v\} \subset \mathbb{N}_0$ is denoted as $\mathbb{I}_s^v$.