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Brief paper

Boundary feedback stabilization of a flexible wing model under unsteady aerodynamic loads ☆

Hugo Lhachemi, David Saussié, Guchuan Zhu *

École Polytechnique de Montréal, Montréal, Canada

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ABSTRACT

This paper addresses the boundary stabilization of a flexible wing model, both in bending and twisting displacements, under unsteady aerodynamic loads, and in presence of a store. The wing dynamics is captured by a distributed parameter system as a coupled Euler–Bernoulli and Timoshenko beam model. The problem is tackled in the framework of semigroup theory, and a Lyapunov-based stability analysis is carried out to assess that the system energy, as well as the bending and twisting displacements, decay exponentially to zero. The effectiveness of the proposed boundary control scheme is evaluated based on simulations.

1. Introduction

Modern aerospace systems such as aircraft, unmanned aerial vehicles (UAVs), and microaerial vehicles are subject to stringent performance requirements including high maneuverability and extended autonomy. The global trend to achieve the required level of performance consists in reducing the mass of the system by a massive integration of composite materials. However, it results in a decrease of the structure rigidity. In particular, lightweight flexible wings are subject to stronger aeroelastic phenomena which are the result from interactions between aerodynamic, elastic and inertial forces. Such phenomena can significantly degrade the performance of an aircraft by introducing undesired couplings between the flexible modes and the flight dynamics (Shearer & Cesnik, 2007; Su & Cesnik, 2010), and may also jeopardize the integrity of its structure (Mukhopadhyay, 2003). These phenomena can be amplified in the case of a store located under the wing with the emergence of the so-called store-induced oscillations (Beran, Strganac, Kim, & Nichkawde, 2004; Bialy, Chakraborty, Cekic, & Dixon, 2016). Therefore, the active control of aeroelastic phenomena has become a topic of primary interest.

One of the most noticeable contributions for the control of aeroelastic phenomena is the Benchmark Active Control Technology (BACT) wind-tunnel model developed by NASA Langley Research Center (Scott, Hoadley, Wieseman, & Durham, 2000). The BACT is modeled as a two-degree-of-freedom aeroelastic wing section capturing the first bending and twisting modes of a flexible wing. The control design strategy of the BACT for flutter suppression, including experimental tests, has been widely investigated in the literature (Bhoir & Singh, 2004; Ko, Strganac, & Kurdila, 1999; Mukhopadhyay, 2000). Nevertheless, the BACT cannot fully represent the dynamics of real flexible wings. Indeed, the flexible wing can be more accurately modeled by a distributed parameter system of two coupled partial differential equations (PDEs) describing the dynamics in bending and twisting displacements respectively (Bialy et al., 2016; Zhang, Xu, Nair, & Chellaboina, 2005; Ziabari & Ghadiri, 2010).

The study on flexible structures described by distributed systems and their interactions with the flow-field has attracted many attention in the last decades (Stanewsky, 2001). The bending dynamics of a panel evolving in different flow-field regimes have been studied for clamped (Chueshov & Lasiecka, 2012; Lasiecka & Webster, 2016) and clamped-free (Chueshov, Dowell, Lasiecka, & Webster, 2016) boundary conditions in case of a distributed velocity feedback. The coupled Euler–Bernoulli and Timoshenko beam model, describing both undamped bending and torsion flexible displacements, has also been investigated for self-straining actuators employed as boundary control inputs without (Balakrishnan, Shubov, & Peterson, 2004) and with (Balakrishnan, 2001, 2003) an external load generated by the flow-field.

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* Corresponding author.
E-mail addresses: hugo.lhachemi@polymtl.ca (H. Lhachemi), d.saussie@polymtl.ca (D. Saussié), guchuan.zhu@polymtl.ca (G. Zhu).

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This paper addresses the boundary stabilization problem of a flexible wing whose dynamics are captured by a coupled Euler–Bernoulli and Timoshenko beam model in the presence of a store located at the wing tip. Unlike the self-straining actuation setup considered in Balakrishnan (2001, 2003) and Balakrishnan et al. (2004), the actuation scheme consists in flaps located at the wing tip to locally generate lift force and torsional momentum, resulting in distinct boundary conditions of the coupled PDEs. Furthermore, the model considered in this work includes the contribution of the Kelvin–Voigt damping (Zhang & Guo, 2011) in both bending and twisting axes. This model is a linear and damped version of the one presented in Bialy et al. (2016), while the aerodynamic loads are supposed to be unsteady. A similar problem, namely a flapping wing UAV, is considered in Paranjape, Guan, Chung, and Krstic (2013). The model used includes the contribution of the Kelvin–Voigt damping, while assuming that aerodynamic loads are unknown but bounded. The method of backstepping is used for the boundary control of the spatial integral of the state variables to track the net aerodynamic forces on the wing. The same model is considered in He and Zhang (2017) for which a Lyapunov-based stabilization control is developed to achieve bounded bending and twisting deflections in the presence of aerodynamic load disturbances. It is worth noting that as pointed out in Curtain and Morris (2009), an Euler–Bernoulli beam model with Kelvin–Voigt damping may not be well-posed if the boundary conditions do not explicitly include the Kelvin–Voigt damping term in a correct manner. Therefore, although the existence of Kelvin–Voigt damping may intuitively be helpful for system stabilization, a rigorous well-posedness analysis is still needed to guarantee the expected behavior of the considered system which remains a more complex setting than a single beam. This constitutes one of the main motivations of the present work.

It should be noticed that a commonly used assumption for Lyapunov-based designs in the works (Bialy et al., 2016; He & Zhang, 2017; Paranjape et al., 2013) is that either the system energy or the aerodynamic loads should be bounded. Furthermore it is also assumed the existence and the regularity of the system trajectories and their partial derivatives up to a certain order. These assumptions, which can be justified by physical intuitions (De Queiroz, Dawson, Nagarkatti, & Zhang, 2012; De Queiroz & Rahn, 2002), can considerably simplify closed-loop stability analysis. However, they imply the well-posedness of the underlying PDEs, which is a quite strong condition. The main objective of this work is to show that these assumptions can be relaxed. To this aim, we formulate the problem under an abstract form that allows the application of the semigroup theory (Curtain & Zwart, 2012; Pazy, 2012). Due to the presence of the store, the boundary conditions related to the control strategy take the form of ODEs (Guo, 2002). It results in a system in abstract form composed of two coupled PDEs and two coupled ODEs. We show that the closed-loop system with the proposed boundary control admits a C_0-semigroup and is well-posed. The closed-loop stability is derived from a Lyapunov-based analysis, which shows that the above C_0-semigroup is exponentially stable. The results of this work allow confirming the validity of most existing control schemes reported in the literature for similar settings under even much less restrictive conditions.

The remainder of the paper is organized as follows. The wing model, along with the associated abstract form, are introduced in Section 2. The well-posedness of the problem is analyzed in Section 3 in the framework of semigroup theory. Then, a Lyapunov-based analysis is carried out in Section 4 to assess that the system energy, as well as bending and twisting displacements, exponentially decay to zero. Finally, numerical simulations are presented in Section 5 to illustrate the performance of the closed-loop system.

Notations (Leoni, 2009; Royden & Fitzpatrick, 1988): $\mathbb{R}_+$ and $\mathbb{R}_+^*$ denote the sets of non-negative and positive real numbers, respectively. Let $L^2(0,l)$ be the set of Lebesgue squared integrable real-valued functions over $(0,l)$ endowed with its natural norm denoted by $\|\cdot\|_{L^2(0,l)}$. For any $m \in \mathbb{N}$, $H^m(0,l)$ denotes the usual Sobolev space, which is defined as the set of $f \in L^2(0,l)$, such that f admits m successive weak derivatives, denoted by $f', f'', \ldots, f^{(m)}$, in $L^2(0,l)$. Denoting by $\mathrm{AC}[0,l]$ the set of all absolutely continuous functions on $[0,l]$, $H^1(0,l) \subset \mathrm{AC}[0,l]$ in the sense that for any $f \in H^1(0,l)$, there exists a unique absolutely continuous function $g \in \mathrm{AC}[0,l]$ such that $f = g$ in $H^1(0,l)$. We note $H_L^m(0,l) = \{f \in H^m(0,l) \;:\; f(0) = f'(0) = \cdots = f^{(m-1)}(0) = 0\}$. For a given normed vector spaces $(E, \|\cdot\|_E)$, $\mathscr{L}(E)$ denotes the space of bounded linear transformations from E to E. The range of a given operator $\mathscr{A}$ is denoted by $R(\mathscr{A})$ while its resolvent set is denoted by $\rho(\mathscr{A})$. The successive partial derivatives of a sufficiently regular function f are denoted in subscript, e.g., f_{ty} stands for $\partial^2 f/(\partial t \partial y)$.

2. Problem setting and boundary control law

2.1. Flexible wing model

Let $l \in \mathbb{R}_+^*$ be the length of the wing, $\rho \in \mathbb{R}_+^*$ the mass per unit of span, $I_w \in \mathbb{R}_+^*$ the moment of inertia per unit length, $EI \in \mathbb{R}_+^*$ (resp. $GJ \in \mathbb{R}_+^*$) the bending (resp. torsional) stiffness, $\eta_\omega \in \mathbb{R}_+^*$ (resp. $\eta_\phi \in \mathbb{R}_+^*$) the bending (resp. torsional) Kelvin–Voigt damping coefficient, and $x_c \in \mathbb{R}$ the distance between the wing center of gravity and the elastic axis of the wing. The store at the wing tip is characterized by its mass $m_s \in \mathbb{R}_+^*$ and its moment of inertia $J_s \in \mathbb{R}_+^*$. We define the two following symmetric definite positive matrices:

$$M \triangleq \begin{bmatrix} \rho & \rho x_c \\ \rho x_c & I_w^* \end{bmatrix}, \qquad M_s \triangleq \begin{bmatrix} m_s & m_s x_c \\ m_s x_c & J_s^* \end{bmatrix}, \tag{1}$$

with $I_w^* \triangleq I_w + \rho x_c^2$ and $J_s^* \triangleq J_s + m_s x_c^2$. Introducing $c_\omega = \sqrt{EI/\rho}$ and $c_\phi = \sqrt{GJ/I_w}$, the bending and twisting dynamics are described by the following coupled PDEs (Bialy et al., 2016; He & Zhang, 2017; Paranjape et al., 2013):

$$M \begin{bmatrix} \omega_{tt} \\ \phi_{tt} \end{bmatrix} + \begin{bmatrix} \rho c_\omega^2 (\omega_{yy} + \eta_\omega \omega_{tyy})_{yy} \\ -I_w c_\phi^2 (\phi_y + \eta_\phi \phi_{ty})_y \end{bmatrix} = \begin{bmatrix} F_a \\ M_a \end{bmatrix}, \tag{2}$$

where the functions $\omega : [0,l] \times \mathbb{R}_+ \to \mathbb{R}$ and $\phi : [0,l] \times \mathbb{R}_+ \to \mathbb{R}$ denote, respectively, the bending and twisting displacements at the location $y \in [0,l]$ along the wing span and at time $t \geq 0$ and $F_a : [0,l] \times \mathbb{R}_+ \to \mathbb{R}$ and $M_a : [0,l] \times \mathbb{R}_+ \to \mathbb{R}$ denote, respectively, the aerodynamic lift force and pitching moment applied at the location $y \in [0,l]$ and at time $t \geq 0$. They are expressed under the following unsteady form:

$$\begin{bmatrix} F_a \\ M_a \end{bmatrix} \triangleq \begin{bmatrix} \alpha_\omega \phi + \beta_\omega \phi_t + \gamma_\omega \omega_t \\ \alpha_\phi \phi + \beta_\phi \phi_t + \gamma_\phi \omega_t \end{bmatrix}, \tag{3}$$

where $\alpha_\omega, \beta_\omega, \gamma_\omega, \alpha_\phi, \beta_\phi, \gamma_\phi \in \mathbb{R}_+$. This model, commonly employed in finite dimension (Bhoir & Singh, 2004; Ko et al., 1999; Mukhopadhyay, 2000), is a trade-off between the used steady form of Bialy et al. (2016) and the unmodeled black-box representation in He and Zhang (2017) and Paranjape et al. (2013). The boundary conditions for the tip-based control scheme, in the presence of a store (Bialy et al., 2016), considered in this work are such that, for any $t \geq 0$,

$$\omega(0,t) = \omega_y(0,t) = \omega_{yy}(l,t) = \phi(0,t) = 0, \tag{4a}$$

$$M_s \begin{bmatrix} \omega_{tt}(l,t) \\ \phi_{tt}(l,t) \end{bmatrix} = \begin{bmatrix} L_{\mathrm{tip}}(t) + \rho c_\omega^2 (\omega_{yy} + \eta_\omega \omega_{tyy})_y (l,t) \\ M_{\mathrm{tip}}(t) - I_w c_\phi^2 (\phi_y + \eta_\phi \phi_{ty})(l,t) \end{bmatrix}, \tag{4b}$$

where $L_{\mathrm{tip}} : \mathbb{R}_+ \to \mathbb{R}$ and $M_{\mathrm{tip}} : \mathbb{R}_+ \to \mathbb{R}$ are the tip control inputs. More precisely, $L_{\mathrm{tip}}(t)$ and $M_{\mathrm{tip}}(t)$ denote the aerodynamic lift force and pitching moment generated at time t by the flaps located at the wing tip. Finally, the initial conditions are given, for any $y \in (0,l)$,