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Brief paper

Joint state and parameter estimation of non-linearly parameterized discrete-time nonlinear systems ☆

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ABSTRACT

Simultaneous state and parameter estimation of nonlinear discrete-time systems is addressed. The developed procedure is dedicated to Multiple-Input–Multiple-Output (MIMO) bounded-state nonlinear systems subject to either linear or nonlinear parametrization. It is shown that under the persistent excitation condition the unmeasured states along with the system parameters can be reproduced asymptotically if the upper and the lower bounds of the system states are assumed to be known. The existence of the observer gain is conditioned by the solvability of a linear matrix inequality (LMI). Simulation results are provided to highlight the efficacy of the developed theoretical results.

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1. Introduction

Joint state and parameter estimation has been the subject of many research papers. Generally, an adaptive observer is referred to as a state estimator that could reproduce the unmeasured states and the system parameters under some matching or persistent-excitation conditions. Early contributions to the theory of adaptive observers can be traced in the Refs. Kreisselmeier (1977) and Lüders and Narendra (1973) where numerous types of dynamical systems are studied. For nonlinear systems, that can be linearized by coordinate transformations, it was shown that the global-state and parameter estimation is possible, see e.g., Besançon, León-Morales, and Huerta-Guevara (2006), Farza, M'Saad, Maatoug, and Kamoun (2009), Marino and Tomei (1995) and the Refs. therein. In Cho and Rajamani (1997) a globally-converging adaptive observer is proposed for Lipschitz systems verifying a matching condition which is somehow restrictive in the general case. An interesting framework for simultaneous state and parameter estimation is proposed in Zhang (2002). Subsequently, the results presented in Zhang (2002) have been exploited to deal with Lipschitz nonlinear systems written in triangular form Zhang and Xu (2001). Recently, an adaptive continuous-time observer for a class of nonlinear parameterized systems has been discussed in the Ref. Farza et al. (2009). A hybrid adaptive observer for a class of nonlinear systems with sampled-data measurements is proposed in Farza, Bouraoui, Ménard, Abdennour, and M'Saad (2014). Adaptive observers and parameter estimation for a class of systems nonlinear in the parameters are discussed in Tyukin, Steur, Nijmeijer, and van Leeuwenb (2013) where a matching condition is required for the estimation process. In Zhao and Hua (2017), a continuous-discrete-time adaptive observer design for a class of nonlinear systems with unknown constant parameters and sampled output measurements is presented. In Ménard et al. (2014), the authors generalize the design proposed in Farza et al. (2009) to a larger class of nonlinear systems. The nonlinear adaptive observer is used in Maouche, MSaad, Bensaker, and Farza (2015) to estimate the mechanical and the magnetic state variables as well as the stator and the rotor resistances of induction motors. Adaptive estimation of a class of deterministic nonlinear time-delay systems is investigated in Ibrir (2009).

Nonlinear observers are ubiquitous in process monitoring, fault detection, isolation, parameter estimation, and control system design, see e.g., Besançon (2007), Hammouri, Kinnaert, and Yaagoubi (1999) and Ibrir (2011). However, irrespective of the observer analysis and synthesis method, the implementation of dynamical models in target computers necessitate the discretization of continuous models to fit with the discrete nature of digital computers and measurements. Furthermore, in many situations, the systems under consideration do not need to be discretized because they appear naturally as pure discrete systems. For further detail on discretization of nonlinear systems, the reader is referred to Kazantzis

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and Kravaris (1999), Monaco and Normand-Cyrot (1984), Nešić and Teel (2001) and the references therein. The reader is also referred to Ren and Guo (2005) for more detail on impossibility design and constraint in discrete time. To the best of our knowledge, adaptive observer design for MIMO discrete-time nonlinear systems subject to nonlinear parametrization was not discussed in the literature; specifically when the system dynamics is not given in a certain canonical form. In contrast to the works published in Farza et al. (2009) and Ménard et al. (2014), that are basically devoted to single-output continuous systems, the proposed adaptive observation method is mainly devoted to MIMO discrete-time nonlinear systems having arbitrary nonlinearity structure. Additionally, the hybrid-observer design given (Farza et al., 2014) is also related to single-output systems. As comparison with adaptive observer design with multiple outputs discussed in Zhao and Hua (2017), the parametrization in that paper is assumed to be linear with respect to the state and the input while the system dynamics is considered as continuous. It is also important to mention that the proposed design is free from additional constraints known as the matching conditions as seen in the Refs. Cho and Rajamani (1997) and Tyukin et al. (2013).

In this paper, the works presented in Farza et al. (2009) and Ménard et al. (2014) are extended to discrete-time nonlinear systems whose dynamics are not necessarily given in some canonical forms. Additionally, the proposed algorithm is conceived to deal with MIMO bounded-state systems using convex-optimization approach. Similar to the continuous-time case, the convergence of the adaptive observer is guaranteed under the persistent excitation condition.

Throughout this paper, $\mathbb{R}$ and $\mathbb{Z}$ denote the set of real and integer numbers, respectively. The notation $A > 0$ (resp. $A < 0$) means that the matrix A is positive definite (resp. negative definite). I is the identity matrix of appropriate dimension and $\mathbf{0}$ stands for the null matrix of appropriate dimension. A' denotes the matrix transpose of A. We note by $\triangleq$ any equality by definition. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ stands for the largest and the lowest eigenvalue of A, respectively. The convex hull of two given vectors x and y is defined as $\mathscr{C}_0(x, y) \triangleq \lambda x + (1-\lambda)y$, $0 \le \lambda \le 1$. The notation $\|\cdot\|$ and $\|\cdot\|_\infty$ stand for the classical Euclidean norm and the infinity norm. For sake of clarity of the main results of this paper, the following results are recalled.

Lemma 1 (*The Schur Complement Lemma Boyd, Ghaoui, Feron, & Balakrishnan, 1994*)**.** *Given constant matrices M, N, Q of appropriate dimensions where M and Q are symmetric, then $Q > 0$ and $M + N'Q^{-1}N < 0$ if and only if*

$$\begin{bmatrix} M & N' \\ N & -Q \end{bmatrix} < 0, \text{ or equivalently } \begin{bmatrix} -Q & N \\ N' & M \end{bmatrix} < 0.$$

Lemma 2 (*The S-Procedure Lemma Boyd et al., 1994*)**.** *Let $F_0, \ldots, F_p$ be quadratic functions of the variables $\zeta \in \mathbb{R}^n$. Consider the following conditions on $F_0, \ldots, F_p$:*

$$F_0(\zeta) \ge 0; \text{ for all } \zeta \text{ such that } F_i(\zeta) \ge 0, \quad 1 \le i \le p. \tag{1}$$

If there exist $\tau_1 \ge 0, \ldots, \tau_p \ge 0$ such that for all ζ, $F_0(\zeta) - \sum_{i=1}^{p} \tau_i F_i(\zeta) \ge 0$ then, (1) *holds.*

The theory of the adaptive observer is given in the following section.

2. Adaptive observer design

2.1. System description

Consider the uncertain discrete-time system governed by the following recurrent equations:

$$\begin{aligned} x_{k+1} &= A x_k + f(x_k, u_k, \theta) + \varphi(u_k, y_k), \\ y_k &= C x_k, \end{aligned} \tag{2}$$

where $k \in \mathbb{Z}_{\ge 0}$ is the time index, (A, C) is an arbitrary observable pair, $x_k = \begin{pmatrix} x_1(k) & x_2(k) & \cdots & x_n(k) \end{pmatrix}' \in \mathscr{M} \subset \mathbb{R}^n$ is the state vector, $u_k \in \mathscr{U} \subset \mathbb{R}^m$ is the control input, $y_k \in \mathbb{R}^p$ is the system output, and $\theta \in \Theta \subset \mathbb{R}^q$ is a vector of unknown, positive, and constant parameters with Θ being a convex set. The nonlinearity $f(x_k, u_k, \theta) : \mathscr{M} \times \mathscr{U} \times \Theta \longmapsto \mathbb{R}^n$ is supposed to be smooth with $f(0, 0, \theta) = 0$. The input–output injection vector $\varphi(u_k, y_k) : \mathscr{U} \times \mathscr{M} \longmapsto \mathbb{R}^n$ is assumed to be measured for all $k \in \mathbb{Z}_{\ge 0}$. The input u_k is said "universal" on $\{0, 1, \ldots, N-1\}$ if it distinguishes every initial states $(x_0, \tilde{x}_0)$ on $\{0, 1, \ldots, N-1\}$. System (2) is said to be "uniformly observable" if every admissible control u_k defined on $\{0, 1, \ldots, N-1\}$, is a universal one. We shall call $\mathscr{U}$ the set of all admissible control inputs that makes system (2) uniformly observable. To complete the system description, the following assumptions are taken into consideration.

Assumption 1. The system input u_k is universal and bounded for all $k \in \mathbb{Z}_{\ge 0}$.

Assumption 2. For a given bounded input $u_k \in \mathscr{U}$ and some initial condition $x_0 \in \mathbb{R}^n$, the state vector x_k does not leave the subset $\mathscr{M} \subset \mathbb{R}^n$ where $\mathscr{M} \triangleq \{x_k \in \mathbb{R}^n,\ k \in \mathbb{Z}_{\ge 0},\ |\ \underline{r}_i \le x_i(k) \le \bar{r}_i,\ 1 \le i \le n\}$ where $(\bar{r}_i)_{1 \le i \le n}$ and $(\underline{r}_i)_{1 \le i \le n}$ are well-known reals.

Assumption 3. The entries of the vector $\theta \in \mathbb{R}^q$ are supposed to be constant, positive and unknown. In addition, it is supposed that there exists a set of constants $\theta_i^{\min}, \theta_i^{\max}$; $1 \le i \le q$ such that

$$\theta_i^{\min} \le \theta_i \le \theta_i^{\max}, \quad 1 \le i \le q. \tag{3}$$

Assumption 4. For $x_k \in \mathscr{M} \subset \mathbb{R}^n$, and for all bounded inputs $u_k \in \mathscr{U}$, there exists a matrix G with $G'G$ being diagonal such that the following holds:

$$\nabla' f_x(x_k, u_k, \theta) \nabla f_x(x_k, u_k, \theta) \le G'G \tag{4}$$

where

$$\nabla f_x(x_k, u_k, \theta) \triangleq \frac{\partial f}{\partial x_k}(x_k, u_k, \theta) = \begin{pmatrix} \partial f_1/\partial x_1 & \partial f_1/\partial x_2 & \cdots & \partial f_1/\partial x_n \\ \partial f_2/\partial x_1 & \partial f_2/\partial x_2 & \cdots & \partial f_2/\partial x_n \\ \vdots & & & \\ \partial f_n/\partial x_1 & \partial f_n/\partial x_2 & \cdots & \partial f_n/\partial x_n \end{pmatrix}. \tag{5}$$

Assumption 5. For given $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2 \in \Theta \subset \mathbb{R}^q$, there exists a real matrix G_θ such that the following holds:

$$\nabla' f_\theta(x_k, u_k, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2) \nabla f_\theta(x_k, u_k, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2) \le G_\theta' G_\theta \tag{6}$$

for all $x_k \in \mathscr{M}$ and all bounded inputs $u_k \in \mathscr{U}$ where

$$\nabla f_\theta(x_k, u_k, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2) \triangleq \frac{\partial f}{\partial \theta}(x_k, u_k, \boldsymbol{\theta}_1) - \frac{\partial f}{\partial \theta}(x_k, u_k, \boldsymbol{\theta}_2). \tag{7}$$

The boundedness of the system states is seen as a natural assumption in many observation exercises. Additionally, many physical systems may also enjoy the property of being input to state stable. It is important to stress that Assumption 4 describes the Lipschitz condition of the system nonlinearity vector $f(x_k, u_k, \theta)$ when the system states are bounded. However, the nonlinearity $f(x_k, u_k, \theta)$ may not be globally Lipschitz for all $x_k \in \mathbb{R}^n$. As a matter of fact, the condition (4) is qualified as a less conservative condition when compared to the standard Lipschitz condition: $\|f(x_k, u_k, \theta) - f(z_k, u_k, \theta)\| \le \gamma_L \|x_k - z_k\|$; for all $x_k \in \mathbb{R}^n$, $z_k \in \mathbb{R}^n$ with γ_L being the Lipschitz constant; generally evaluated as $\|\partial f(x_k, u_k, \theta)/\partial x_k\|_\infty$. This comes from the fact that the norm of the Jacobian is not used in (4) to characterize the boundedness of

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