Electric Power Systems Research 164 (2018) 253–262

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Modeling of current distribution in zinc oxide varistors using Voronoi network and finite element method

Zumret Topcagic[a,1,*], Thomas Tsovilis[a,2], Dejan Krizaj[b,3]

[a] Raycap Group, Stegne 23, Ljubljana 1000, Slovenia
[b] University of Ljubljana, Faculty of Electrical Engineering, Trzaska c. 25, 1000 Ljubljana, Slovenia

ARTICLE INFO

Keywords:
Varistor
Finite element method (FEM)
Voronoi network
Current distribution
Non-uniformity
Current localization factor
SPDs

ABSTRACT

Nonuniform current distribution inside varistor ceramics is a key factor influencing its performance and failures. Therefore understanding, modeling and predicting of current distribution in varistor ceramics is of crucial significance. This paper proposes a numerical model for simulation of nonuniform electric current distribution inside zinc oxide varistors. A numerical model is based on physical modeling of the varistor's grain-structured geometry presented by Voronoi network using finite element method (FEM) simulation. The presented method is solving complete electric field inside the modeled geometry and therefore provides a more physically accurate approach for better understanding and predicting nonuniform current distribution in the varistor. In order to properly establish a FEM model a novel approach in defining grain boundary characteristic is proposed. Thus, a macroscopic model of the varistor microstructure has been developed and the grain micro-junction boundary characteristic has been derived. The simulation results of nonuniform current distribution in a varistor agree well with measurement results for a typical ZnO varistor. The presented model enables investigation of influences of varistor geometry (shapes, sizes) and material properties on the current distribution. A new mathematical expression for varistor *I–V* characteristic based on Lambert function is proposed.

1. Introduction

The groundwork of zinc oxide varistors was made in 1968 and later presented by Matsuoka in [1], where the author describes zinc oxide ceramics in detail, production of it, temperature characteristics, nonlinear properties, etc. Shortly after, the zinc oxide varistors were commercialized and have become the most dominant surge protection element in the industry [2]. Due to the grain microstructured nature of zinc oxide ceramics, distribution of additives and production process, the non-uniformity became an inherited property of any ZnO varistor. First to explore non-uniformity phenomena where Mizukoshi et al. [3]. Following their results Eda in [4] discussed puncture failure of ZnO varistors due to current localization caused by varistor non-uniformity. Structural non-uniformity of ZnO varistor was proven to be a key factor causing current localization inside of the varistor ceramics. Pike et al. [5] have experimentally shown current localization at microstructure by means of electroluminescence. The fact that origins of "electrical" non-uniformity of ZnO varistor are not only caused by structural non-uniformity such as grains size distribution has been proven by Tao et al. [6]. The authors have experimentally shown that every single micro-junction in varistor ceramics presents its own electrical characteristic. Therefore, electric current distribution inside the ZnO varistor is determined by structural non-uniformity, distribution of electrical properties of single micro-junctions and, as shown in [3,7] as well as confirmed in this work, on the applied voltage across the varistor. Vojta et al. [8] have simulated the effect of current localization in varistor ceramics by proposing a uniform network of nonlinear resistors as a model of the grain microstructure. To more accurately represent non-uniformity of the varistor ceramics Bartkowiak et al. [9] proposed a Voronoi network as a model of the varistor microstructure. Comparing the Voronoi network geometry and scanning electron microscope (SEM) imaging of ZnO crystals as shown in [10,11], one can easily justify the use of such approach. In [9] authors have used the Voronoi network as a basis for constructing a non-uniformly connected network of nonlinear resistors which (as also shown in [8]) results in solving a set of current conservation equations by means of circuit analysis. By

---

[*] Corresponding author.
E-mail addresses: zumret.topcagic@gmail.com (Z. Topcagic), tsovilis@ieee.org (T. Tsovilis), dejan.krizaj@fe.uni-lj.si (D. Krizaj).
[1] Z. Topcagic is with the R&D Department of Raycap Group, Ljubljana, Slovenia.
[2] T. Tsovilis is R&D Director of Raycap Group, Ljubljana, Slovenia.
[3] D. Krizaj is with the Faculty of Electrical Engineering, University of Ljubljana.

https://doi.org/10.1016/j.epsr.2018.08.001
Received 8 January 2018; Received in revised form 4 July 2018; Accepted 1 August 2018
0378-7796/

Nomenclature

A_{var}	Varistor surface area, m^2
b_1, b_2, b_3, b_4	EPCOS IV characteristic model fitting parameters,
$\overrightarrow{E}$	Electric field, V/m
G	Varistor conductance, S
h_{var}	Varistor height (thickness), m
I	Electric current, A
I_{grain}	Single grain current, A
I_{90}	Varistor current equal to 90% of total current, A
$\overrightarrow{J}$	Current density, A/m^2
k	Number of grains carrying 90% of total current,
N	Number of grains in the numerical model,
N_v	Number of grains in a voltage-applied (vertical) direction,
n_J	Current density localization factor,
n_I	Grain current localization factor,
n_e	Electrode current localization factor,
p	Grain participation factor, %
R_s	Varistor series resistance evaluated at surge region, Ω
ρ_b	Grain contact surface resistance, Ωm^2
ρ_g	Specific grain resistance, Ωm
ρ_l	Grain contact leakage surface resistance, Ωm^2
R_{var}	Varistor resistance, Ω
σ	Specific conductance, S/m
V	Electric potential, V
V_b	Grain boundary voltage, V
V_{th}	Varistor threshold (knee) voltage, V
$W(x)$	Lambert function,
θ, ζ, θ_1, θ_2, ζ_1, ζ_2	Fitting parameters

assigning different (distributed) threshold voltages and nonlinearity coefficients to the nonlinear resistors in the Voronoi generated network the authors in [9] managed to combine both structural and “electrical” non-uniformity of varistor ceramics. After this work, several contributions have been reported [11,12,7,13] using numerical models with Voronoi network for analysis of ZnO varistor electrical characteristics. Current non-uniformity modeling inside ZnO varistor using Voronoi network presented by He et al. [7] showed current localization phenomena dependency on the applied voltage across the varistor. The results indicated that localization reaches its maximum at TOV (temporary overvoltage) [14] region of voltage amplitudes. Modeling of ZnO varistor with finite element method (FEM) was previously reported by Lengauer et al. [15]. They performed FEM analysis to calculate mechanical stresses inside a ZnO disk where the disk ceramics was represented as a homogeneous solid cylinder. Frigura-Iliasa et al. [16] presented a FEM model for calculation of heat losses in the varistor by assuming varistor ceramics as a homogeneous solid (as in [15]). Bavelis et al. [17] proposed a 3D FEM model for current distribution analysis inside a ZnO varistor by representing the grain-structured geometry with a 3D Voronoi network. A drawback of this work is that the FEM model was used only to calculate a conductance matrix of ZnO grains which was later used in a 3D nonlinear resistor network. Therefore the model presented in [17] still results in a circuit analysis without solving for the electric field in the modeled geometry.

In this work, we propose a combination of a Voronoi network and a FEM for modeling current distribution and other physical phenomena inside the ZnO varistor. In presented model Voronoi network geometry representing actual grains with geometrical, material and boundary properties is imported into FEM environment and solved for the electric field and electric current distribution. In order to properly establish a FEM model a novel approach in defining the grain boundary characteristic (condition) is proposed resulting in an accurate simulation of actual *I*–*V* characteristics of the modeled varistor. In order to derive the grain micro-junction boundary characteristic, a macroscopic model of the varistor microstructure has been developed. For confirmation of the suitability of the proposed model a comparison of modeled current localization with measured data has been performed. An estimation of current localization based on a modified approach from [8] is made. In conclusion, we discuss further possibilities of the developed FEM based model and the usage of Lambert function [18] as a basis for varistor *I*–*V* characteristic models.

2. FEM-Voronoi model

2.1. Geometry

The varistor for which the current distribution and localization factor will be calculated and eventually measured was V300S40 from manufacturer Varsi [19]. From this varistor, the generalization of grain boundary characteristic was also derived. This varistor type has been chosen as a reference as it is a typical varistor that is most commonly used in low voltage systems. The V300S40 varistor is integrated to the Raycap Class II SPDs installed to 230/400 V and 240/415 V 3WYE systems. It is a square shaped disk with surface area A_{var} = 16 cm^2, height h_{var} = 3 mm, voltage at 1 mA ($V_{1\text{mA}}$) hereinafter referred as a threshold voltage V_{th} = 500 V and current ratting I_n = 20 kA.[4] Since random nature of varistor's grain structured ceramics excludes symmetries in any direction in principle a 3D Voronoi network [17] and FEM model should be used. Presented mathematical approach and derived boundary conditions are remaining exactly the same in case of 3D Voronoi network. However, such model would demand more computation time and cause problems with calculation and convergence. The issue of convergence is most critical due to the high nonlinearity of derived boundary conditions. For studying current distribution a 2D numerical models of the varistor microstructure have already been proven to be sufficient by previous reports [9], [11], [7] and also here. Therefore in this report, a 2D model of a varistor microstructure is used where the symmetry in *z* “into the paper” direction is presumed. To further reduce the model size and the corresponding computational time the model width is set to ≈1/3 of an actual varistor. It is worth mentioning that in [9], [11] and some other reports the authors have mainly used 1 × 1 normalized square representing varistor geometry. For selected model dimensions a Voronoi network was generated in Wolfram Mathematica [20] environment by the code shown in Algorithm 1.

Algorithm 1. Mathematica Wolfram code for generating and exporting Voronoi network geometry. RandomReal [20] function is used for generating random seeds in the Voronoi network.

```
N=3000;
X=15;
Y=3;
x=RandomReal[X, N, WorkingPrecision → 6];
y=RandomReal[Y, N, WorkingPrecision → 6];
p=Table[{x[[i]], y[[i]]}, {i, N}];
vn=VoronoiMesh[p, PlotTheme → “Lines”,
PlotRange → {{0, X}, {0, Y}}, Frame → True]
Export[“voronoi_network.dxf”, vn]
```

Geometry generated by Algorithm 1, was then exported to a **.dxf** file and imported into a Comsol Multiphysics [21] environment to perform

---

[4] 8/20 surge amplitude defined according to IEC 61643-11.