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Single shot multiple phase retrieval in digital holographic interferometry using subspace processing

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ABSTRACT

The article presents a method to estimate multiple phases from a single moire fringe pattern in digital holographic interferometry. The proposed method relies on analysing the holographic moire signal using subspace rotational invariance approach and offers feasibility for single shot multi-component estimation without multiple frames or spatial carrier and high performance against severe noise. Simulation and experimental results demonstrate the applicability of the proposed method.

1. Introduction

Non-invasive multi-dimensional deformation analysis is an important but challenging problem in experimental mechanics and non-destructive testing where the reliable measurement of in-plane and out of plane components of displacement and strain of a deformed object is strongly desired. In this domain, digital holographic interferometry (DHI) [1] has emerged as a prominent measurement technique. For these measurements, a multi-wave setup is used where the test object is illuminated from different directions, and the interference between the multiple object and reference waves is recorded [2]. Consequently, the information about multi-dimensional displacement components is encoded in multiple interference phase maps, and reliable extraction of these phases carries great significance. Methods based on the application of multiple reference beams [3] and multiple wavelengths [4] have been proposed for multi-dimensional deformation measurements but the corresponding experimental design and setups are practically complex. The experimental complexity is simplified by the single reference wave and single wavelength based design in digital holographic moire [5].

Conventionally, phase retrieval from recorded interferogram is referred to as fringe analysis [6] and several methods have been reported in literature [7–11]. For multi-component fringe analysis or retrieving information about the multiple phases in digital holography, spatial carrier based approaches [2,5] have been applied where the contribution of individual components is separated in the frequency domain using a carrier followed by spectral filtering operation. However, the main limitation of these approaches is the requirement of careful carrier addition and control. Phase shifting based methods [12,13] have also been reported but they require multiple frames to be captured which is practically tedious, error-prone and unsuitable for dynamic measurements. State-space approach [14] has also been proposed for these measurements but the performance is strongly coupled with precise choice of state initialization parameters and high noise-susceptibility. Recently , product high-order ambiguity function [15] and matrix enhancement [16–18] methods have been outlined but they have high computational cost due to the involvement of peak tracing and large matrix operations.

In this work, we propose an elegant method for single shot multiple phase retrieval in digital holographic interferometry which exhibits high robustness against noise, eliminates the need for having multiple frames or careful carrier control, offers good computational efficiency without involving intensive peak search, enhanced matrices or precise initialization. In addition, the inherent mathematical formulation in this method enables direct multiple phase derivative retrieval which is a significant requirement for measuring multi-dimensional components of strain of a deformed object. The outline of the paper is as follows. The theory of the proposed subspace processing method is presented in Section 2. The simulation and experimental results are outlined in Section 3. This is followed by discussions in Section 4. Finally, the conclusions are presented in Section 5.

2. Theory

In digital holographic interferometry, multiple interference phase maps can be efficiently encoded in a moire field [5]. The optical setup for recording moire signal relies on multi-wave illumination of the test object, which is shown in Fig. 1. Mathematically, a dual component digital holographic moire signal can be expressed as,

$$\mathbf{\Gamma}_m(x,y) = A_1(x,y)e^{j\phi_1(x,y)} + A_2(x,y)e^{j\phi_2(x,y)} + \eta(x,y) \quad (1)$$

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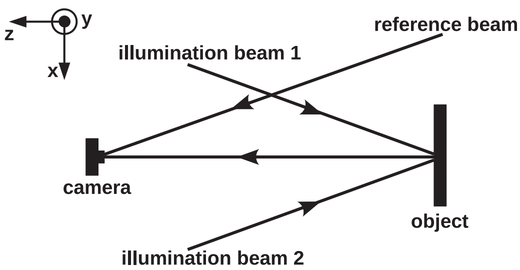

Fig. 1. Schematic for multi-wave illumination.

where A_1 and A_2 are the amplitude terms assumed to be slowly varying, ϕ_1 and ϕ_2 are two interference phases, $y \in [0, N_y - 1]$ and $x \in [0, N_x - 1]$ are the pixels along vertical and horizontal dimensions, and η is complex additive white Gaussian noise (AWGN). In the proposed method, we select a small symmetrical window or block around each pixel (*x, y*) such that the different phase distributions can be modelled as first-order polynomials within the given window. Hence, the windowed moire field can be described in matrix notation as,

$$\Gamma[k,l] = A_1[k,l]e^{j\phi_1[k,l]} + A_2[k,l]e^{j\phi_2[k,l]} + \eta[k,l] \tag{2}$$

where $\phi_1(k,l) = a_0 + a_1 k + a_2 l$, $\phi_2(k,l) = b_0 + b_1 k + b_2 l$, and k and l now indicate the row and column indices such that $k, l \in [-L, L]$ where L is a parameter controlling size of the window. Note that by selecting a small block, the linear phase approximation is valid since the phases are assumed to have slow variations inside the given block. The coefficients $[a_1, a_2]$ effectively represent the spatial frequencies or equivalently the phase derivatives along the two spatial coordinates for the first component, and $[b_1, b_2]$ correspond to the spatial frequencies for the second component. The above signal can be further reduced as

$$\mathbf{\Gamma} = \mathbf{QAR} + \mathbf{W} \tag{3}$$

where

$$\mathbf{Q} = \begin{bmatrix} e^{j(-L)a_1} & e^{j(-L+1)a_1} & \dots & e^{jLa_1} \\ e^{j(-L)b_1} & e^{j(-L+1)b_1} & \dots & e^{jLb_1} \end{bmatrix}^T$$
$$\mathbf{A} = \begin{bmatrix} A_1 e^{ja_0} & 0 \\ 0 & A_2 e^{jb_0} \end{bmatrix}$$
$$\mathbf{R} = \begin{bmatrix} e^{j(-L)a_2} & e^{j(-L+1)a_2} & \dots & e^{jLa_2} \\ e^{j(-L)b_2} & e^{j(-L+1)b_2} & \dots & e^{jLb_2} \end{bmatrix} \tag{4}$$

and **W** represents complex AWGN. The superscript $(.)^T$ denotes transpose of a vector or a matrix.

With $M = 2L + 1$, we define two matrices such that,

$$\begin{aligned} \mathbf{Q_1} &= \begin{bmatrix} \mathbf{I}_{M-1} & \mathbf{0}_{M-1} \end{bmatrix}\mathbf{Q} \\ \mathbf{Q_2} &= \begin{bmatrix} \mathbf{0}_{M-1} & \mathbf{I}_{M-1} \end{bmatrix}\mathbf{Q} \end{aligned} \tag{5}$$

where $\mathbf{I}_{M-1}$ is an identity matrix of order $(M-1) \times (M-1)$ and $\mathbf{0}_{M-1}$ is a zero column matrix of size $(M-1)$. In Eq. 5, the matrices $\mathbf{Q_1}$ and $\mathbf{Q_2}$ are created by truncating last row and first row of **Q**. Consequently, the matrices $\mathbf{Q_1}$ and $\mathbf{Q_2}$ are related as,

$$\mathbf{Q_1} \underbrace{\begin{bmatrix} e^{ja_1} & 0 \\ 0 & e^{jb_1} \end{bmatrix}}_{\mathbf{\Omega_k}} = \mathbf{Q_2} \tag{6}$$

where $\mathbf{\Omega_k}$ is an unitary matrix. We can observe in Eq. 6 that the transformation from $\mathbf{Q_1}$ to $\mathbf{Q_2}$ is rotational (phase change in complex domain) which is a consequence of translational invariance property [19], which signifies the ability to select at least two subsets of the array that have same geometrical shape and size; in this case, one subset has all rows except the last row and the second subset has all rows except the first.

Applying singular value decomposition (SVD) to $\mathbf{\Gamma}$, we get

$$\mathbf{\Gamma}(k,l) = \mathbf{USV} = \sum_{p=0}^{p=M-1} s_p \mathbf{u}_p \mathbf{v}_p \tag{7}$$

where $M = 2L + 1$, **S** is a diagonal matrix with singular values sorted in descending order along the diagonal, **U** contains left singular vectors $\mathbf{u}_p$ and $\mathbf{V}^H$ contains right singular vectors $\mathbf{v}_p^H$ along their columns, $(.)^H$ denotes conjugate transpose of a vector or a matrix. Here, the vectors $\mathbf{u_p}$ represent columns of **U** with size *M*x1 and $\mathbf{v_p}$ denote rows of **V** with size 1x*M*. If the matrix $\mathbf{\Gamma}$ contains *P* sinusoidal signals, then the first *P* singular values and vectors represent signal subspace whereas the remaining singular values and vectors correspond to noise subspace, as shown in the following equation,

$$\mathbf{\Gamma} = \mathbf{U_s S_s V_s} + \mathbf{U_n S_n V_n} \tag{8}$$

where,

$$\begin{aligned} \mathbf{U_s} &= [\mathbf{u}_0 \ \mathbf{u}_1 \ \cdots \ \mathbf{u}_{P-1}] \\ \mathbf{U_n} &= [\mathbf{u}_P \ \mathbf{u}_{P+1} \ \cdots \ \mathbf{u}_{M-1}] \\ \mathbf{V_s} &= [\mathbf{v}_0^T \ \mathbf{v}_1^T \ \cdots \ \mathbf{v}_{P-1}^T]^T \\ \mathbf{V_n} &= [\mathbf{v}_P^T \ \mathbf{v}_{P+1}^T \ \cdots \ \mathbf{v}_{M-1}^T]^T \\ \mathbf{S_s} &= diag(s_0, s_1, \cdots, s_{P-1}) \text{ and} \\ \mathbf{S_n} &= diag(s_P, s_{P+1}, \cdots, s_{M-1}) \end{aligned}$$

The first singular value s_0 corresponds to the strongest signal component whereas s_{P-1} corresponds to the signal component with lowest amplitude. For our analysis, we assume $A_1 > A_2$, and the amplitude discrimination is achieved between the components experimentally by placing a neutral density filter in the path of one of the object waves. In the absence of noise, all singular values representing noise subspace become zero. Consequently, for the case of two signal components, comparing Eqs. (3) and (8) we can say that the matrices **Q** and $\mathbf{U_s}$ have same column space whereas **R** and $\mathbf{V_s}$ have same row space. Using these properties, the relation between **Q** and $\mathbf{U_s}$ is given by,

$$\mathbf{Q} = \mathbf{U_s T} \tag{9}$$

where **T** is a non-singular matrix and $\mathbf{U_s} = [\mathbf{u_0} \ \mathbf{u_1}]$. Using Eqs. (9) and (6), we get

$$\mathbf{U_{s1} T \Omega_k} = \mathbf{U_{s2} T} \tag{10}$$

where,

$$\begin{aligned} \mathbf{U_{s1}} &= \begin{bmatrix} \mathbf{I}_{M-1} & \mathbf{0}_{M-1} \end{bmatrix}\mathbf{U_s} \\ \mathbf{U_{s2}} &= \begin{bmatrix} \mathbf{0}_{M-1} & \mathbf{I}_{M-1} \end{bmatrix}\mathbf{U_s} \end{aligned}$$

Substituting $\mathbf{\Psi_k} = \mathbf{T\Omega_k T}^{-1}$ in Eq. (10), we get,

$$\mathbf{U_{s1}\Psi_k} = \mathbf{U_{s2}} \tag{11}$$

Note that the matrices $\mathbf{\Psi_k}$ and $\mathbf{\Omega_k}$ are similar matrices and have same eigenvalues. Hence, solving for eigenvalues of matrix $\mathbf{\Psi_k}$ provides information about the spatial frequencies [19]. Note that Eq. (11) represents a system of over-determined linear equations which can be solved using least squares (LS) approach by applying pseudo-inverse operation,

$$\hat{\mathbf{\Psi}}_\mathbf{k} = \underset{\mathbf{\Psi_k}}{\operatorname{argmin}} \ ||\mathbf{U_{s1}\Psi_k} - \mathbf{U_{s2}}||^2 = \mathbf{U_{s1}^+ U_{s2}} \tag{12}$$

where $(.)^+$ denote Moore–Penrose pseudo-inverse operation. Further, in our analysis, the first component is the dominant or stronger one in terms of amplitude, and the first eigen value of $\hat{\mathbf{\Psi}}_\mathbf{k}$ directly corresponds to the spatial frequency or equivalently the first order coefficient of the dominant component, which is estimated as,

$$\hat{a}_1 = arg(\lambda_{k1}) \tag{13}$$

where λ_{k1} is the first eigenvalue of $\hat{\mathbf{\Psi}}_\mathbf{k}$.

Using the same analysis for $\mathbf{V_s}$, we get,

$$\hat{\mathbf{\Psi}}_\mathbf{l} = \underset{\mathbf{\Psi_l}}{\operatorname{argmin}} \ ||\mathbf{V_{s1}\Psi_l} - \mathbf{V_{s2}}||^2 = \mathbf{V_{s1}^+ V_{s2}} \tag{14}$$

where,

$$\mathbf{V_{s1}} = \begin{bmatrix} \mathbf{I}_{M-1} & \mathbf{0}_{M-1} \end{bmatrix}\mathbf{V_s}^T$$