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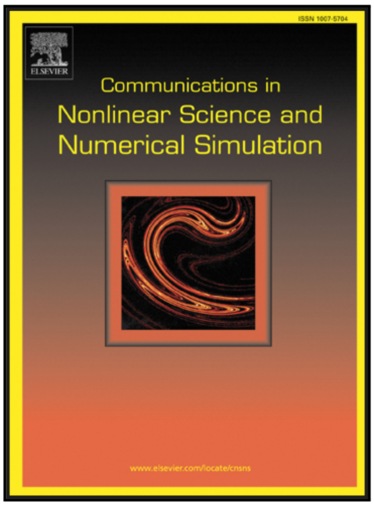

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Torsional instability and sensitivity analysis in a suspension bridge model related to the Melan equation

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Abstract

Inspired by the Melan equation we propose a model for suspension bridges with two cables linked to a deck, through inextensible hangers. We write the energy of the system and we derive from variational principles two nonlinear and nonlocal hyperbolic partial differential equations, involving the vertical displacement and the torsional rotation of the deck. We prove existence and uniqueness of a weak solution and we perform some numerical experiments on the isolated system; moreover we propose a sensitivity analysis of the system by mechanical parameters in terms of torsional instability. Our results display that there are specific thresholds of torsional instability with respect to the initial amplitude of the longitudinal mode excited.

Keywords: suspension bridges, torsional instability, hyperbolic problem, nonlocal term.

1 Introduction

The Melan equation was introduced by the Austrian engineer Josef Melan [16] in 1888 to model a suspension bridge; Melan considered the bridge as a combination of a string (the cable) and a beam (the deck) linked through some rigid hangers, which are considered uniformly distributed along the main span.

The equation can be derived writing the equilibrium of the beam and the string and combining the two equations through the live load, carried in part by the cable and in part by the deck. The result is the following fourth order differential equation

$$\begin{cases} EIw''''(x) - (H + h(w))w''(x) - \frac{q}{H}h(w) = p(x) & \forall x \in (0, L) \\ w(0) = w(L) = w''(0) = w''(L) = 0, \end{cases} \tag{1.1}$$

in which $w(x)$ is the vertical displacement of the beam (positive if directed downward), EI is the flexural rigidity of the beam, H is the horizontal tension of the string when subjected to the dead load $-q$, and $h(w)$ is a nonlocal term, representing the additional tension in the cable due to the live load $p(x)$; the beam has a span equal to L and is supposed hinged at the endpoints. The presence of the nonlocal term makes challenging the study of the equation from both the theoretical as from the numerical point of view, see e.g. [10, 11, 21]; although (1.1) cannot be derived from the variation of the corresponding energy [11], von Kármán-Biot [23] call the Melan equation (1.1) the *fundamental equation of the theory of the suspension bridge.*

This equation is our starting point, we propose a more reliable model for suspension bridge in which we have two strings (the cables) linked to the same deck, through inextensible hangers, see Section 2.1. In this way we introduce the torsional rotation of the deck, which cannot be