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Axisymmetric vibrations of a circular Chladni plate in air and fully submerged in water

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ABSTRACT

In this study, experimental and numerical analyses were performed to determine the effects of water loading on the axisymmetric modes of vibration of a circular plate. The plate was harmonically excited at its centre through an extension bar and its outer edge was left free. The Chladni technique, which involves exciting the plate at a resonance and waiting for sand grains sprinkled on the plate to collect along the nodal circles, was used to identify and visualize the modes both in air and fully submerged in water. Surprisingly, inverse Chladni patterns were observed in water as particles were drawn towards the zero transversal velocity radii by the induced flow. A coupled acoustic–structural finite element model was built to simulate the same modes, which had been preliminarily validated against theoretical results of a completely free edged plate. A good agreement between experimental and numerical natural frequencies and mode shapes was found. The frequency reduction ratio due to the added mass effect was around 64 %. Moreover, measurable differences due to fluid–structure coupling were observed in the radii of the nodal circles between corresponding dry and wet modes.

1. Introduction

Starting with the pioneering work of Lindholm et al. (1965) and Blevins (1979), the fluid–structure interaction (FSI) phenomena has been the focus of many experimental, numerical and theoretical investigations. Recently, FSI problems have received much attention in the field of hydraulics due to the importance of studying the dynamic responses of pipelines and machines, as reported by Li et al. (2015) and Trivedi and Cervantes (2017), respectively. In the case of water turbines, Rodriguez et al. (2006) and Lais et al. (2009) investigated the added mass effects of water on the modal properties of complex structures such as the runners. In this sense, it is well known that the presence of a dense fluid surrounding the solid causes a reduction in the natural frequencies compared to air. In general, identical mode shapes have been assumed for a structures in air and fully submerged in water.

The authors of the present work recently became interested in the dynamic responses of bodies submerged in two-phase flows. For example, the influence of cavitation on the structural response of a hydrofoil (De La Torre et al., 2013, 2014; Liu et al., 2017) and the directional added mass effects in partially liquid-filled horizontal pipes (Escaler et al., 2017) have been experimentally and numerically studied. From these investigations, the authors of this current work have detected that small but relevant differences may exist between the mode shapes in air and in water. This observation agrees with the theoretical results of Amabili et al. (1995), Amabili et al. (1996), Amabili (1996) and Amabili and Kwak (1996) who

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compared the assumed-modes approach and the Rayleigh–Ritz method to calculate the vibrations of annular and circular plates coupled with fluids. The former approach assumes that the mode shapes are not modified by the fluid, i.e. dry and wet mode shapes are exactly the same, meanwhile the latter method removes this simplifying hypothesis. Their calculations showed that with the Rayleigh–Ritz method the nodal circles in water can change around 5% depending on the mode and the boundary conditions. Similarly, Junger and Feit (1986) noted that underwater mode shapes were slightly deformed compared to the mode shapes in air, which was likely due to modal coupling by the dense fluid. Furthermore, the differences appear considerably larger when the body is partially in contact with water and air, as presented in De La Torre et al. (2016).

The present manuscript addresses the experiments and numerical simulations used to quantify the changes in the modes of vibration of a simple structure with the addition of a surrounding fluid. Specifically, the modal vibrations of a circular Chladni plate were studied in air and completely submerged in a water tank. The idea behind this experiment was to excite the axisymmetric modes and to infer the mode shapes from the nodal circles by using the method published by Chladni (1787). Modal changes can be quantified by comparing the results from air and water along with the use of simulations.

The Chladni patterns can be generated by exciting a horizontal plate at a single vibrational mode and then waiting for sand grains scattered on the top surface to collect along the nodal lines. The Chladni technique allows for the observation of vibration modes and has been commonly used to study the frequencies of plates with different geometries and dimensions. To our knowledge, this visualization method has only been used in air, similar to the work of Rossing (1982). Therefore, the intention of this study is to validate the method for a plate submerged in still water.

Regarding alternative experimental modal test methods, Bergen and Pechersky (1991) carried out a similar investigation on submerged composite square plates using Digital Speckle Pattern Interferometry (DSPI), which is similar to holographic interferometry. Askari et al. (2013) conducted modal tests on circular plates immersed in a fluid at various depths; the plate responses were measured with a laser Doppler vibrometer through a transparent wall at the bottom of the tank, and the experimental results were used to validate a theoretical model. To infer a mode shape from the experiment, only 4 or 5 measurements (equally distributed along the radius) were presented for the axisymmetric modes with 1 and 2 nodal circles. In both of these studies, the proposed visualization methods are far more expensive and less practical than the methods used in the present study. The Chladni technique has been proved to visualize the nodal lines with a high spatial resolution.

Recently, several analytical and numerical methods have been proposed to determine the modal characteristics of circular and annular plates in fluid. In particular, Kwak and Amabili (1999) analysed theoretically the natural frequencies of free-edge annular plates fully immersed in water. Jhung et al. (2009) developed an analytical method to assess a perforated plate in contact with or submerged in fluid. Garrido-Mendoza et al. (2013) studied the hydrodynamic coefficients of added mass and damping of an oscillating disk approaching a seabed using OpenFOAM® software. Finally, Gascón-Pérez and García-Fogeda (2015) developed a method to compute the natural frequencies and acoustic damping ratio of a circular plate surrounded by a compressible fluid of arbitrary density. In our study, we have used the acoustic–structural finite element analysis (FEA) tool available in ANSYS® Mechanical, which takes into account the FSI phenomena.

2. Axisymmetric vibrations of a circular plate completely free

Leissa (1969) and Leissa and Narita (1980) calculated the natural frequencies of circular plates in a vacuum by solving the classical differential equation of motion for the transverse displacement of a plate given by

$$D\nabla^4 w + \rho_A \frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

where ρ_A is the area density of the plate, t is the time, $\nabla^4 = \nabla^2\nabla^2$, where ∇^2 is the Laplace operator and D is the flexural rigidity defined by

$$D = \frac{Eh^2}{12\left(1-\nu^2\right)}. \tag{2}$$

For free vibration, the motion can be expressed as

$$w = W\cos\omega t \tag{3}$$

where ω is the angular frequency, and W is a function of the position coordinates. Substituting Eq. (3) into Eq. (1) yields

$$\left(\nabla^4 - k^4\right)W = 0 \tag{4}$$

where k is a parameter of convenience defined as

$$k^4 = \frac{\rho\omega^2}{D} \tag{5}$$

For a solid circular plate (having no internal holes) with the polar coordinate system origin coincident with the centre of the plate, the solution to Eq. (4) for values of n from 0 to ∞ becomes

$$W_n = \left[A_n J_n(kr) + C_n I_n(kr)\right]\cos n\theta \tag{6}$$