Journal of the Mechanics and Physics of Solids 121 (2018) 328–340

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Symmetry breaking during inflation of a toroidal membrane

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ARTICLE INFO

Article history:
Received 21 February 2018
Revised 25 June 2018
Accepted 12 August 2018
Available online 14 August 2018

Keywords:
Toroidal membrane
Geometric instability
Symmetry breaking
Supercritical pitchfork bifurcation
Reverse subcritical pitchfork bifurcation

ABSTRACT

Membrane structures with symmetry often exhibit geometric instability under finite inflation. We observe and study this phenomenon in the case of a flat toroidal membrane with axisymmetry and reflection symmetry. The membrane is modeled as a Mooney-Rivlin hyperelastic material. The stability of the symmetric inflation path is studied perturbatively which reveals a zone of instability. Using higher order perturbation, the asymmetric shape is subsequently determined. As the inflation proceeds, first the axisymmetry is broken spontaneously through a supercritical pitchfork bifurcation, and is later restored at a reverse subcritical pitchfork bifurcation. The extent of the symmetry breaking zone depends on the material and geometric parameters of the toroidal structure. It is found that a stout torus can completely eliminate the occurrence of such symmetry breaking bifurcations on its inflation path.

1. Introduction

Inflatable structures have evoked considerable interest in several modern day applications, such as space structures, vibration isolators, inflatable reflectors, artificial actuators, bio-mechanical and tunable devices (see, e.g., Blum et al. (2011); Carpi et al. (2011); Jenkins (2001, 2006); Xiao et al. (2013)). Typically, in such applications, the structure is an internally pressurized membrane of a certain shape. The stability of such inflated membrane structures under pressurization and external loading is an important issue to be addressed before putting them to use. While occurrence of instability beyond a critical value of external loading is expected, geometric instability leading to a loss of shape/symmetry during finite inflation/loading is more subtle, and requires a careful approach to uncover. The loss of symmetry of an inflatable structure appears as a global instability over the domain of the structure. In the case of a toroidal inflatable membrane, the problem appears to be open. The toroidal structure is one common geometry that has wide applicability, such as supporting rim of inflatable reflectors, antenna, lenses and solar concentrators.

Symmetry in geometry (both axisymmetry and reflection symmetry) is quite common for inflatable structures in nature, as well as in most practical applications. Typically, in the analysis of large inflation/deformation problems (see, e.g., Mooney, 1940; Rivlin, 1948a,b), the choice of the solution ansatz is motivated by the symmetries of the problem. However, due to a complex interplay between geometric and material nonlinearities, under certain conditions, such symmetries of the structure may be broken through a geometric instability. Such symmetry breaking will not be reflected in the theoretical results if they are not accommodated in the solution ansatz. On the other hand, starting with a general formulation amplifies the computational complexity of the problem. Furthermore, in numerical computations based on discretization, unsymmetric

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https://doi.org/10.1016/j.jmps.2018.08.008

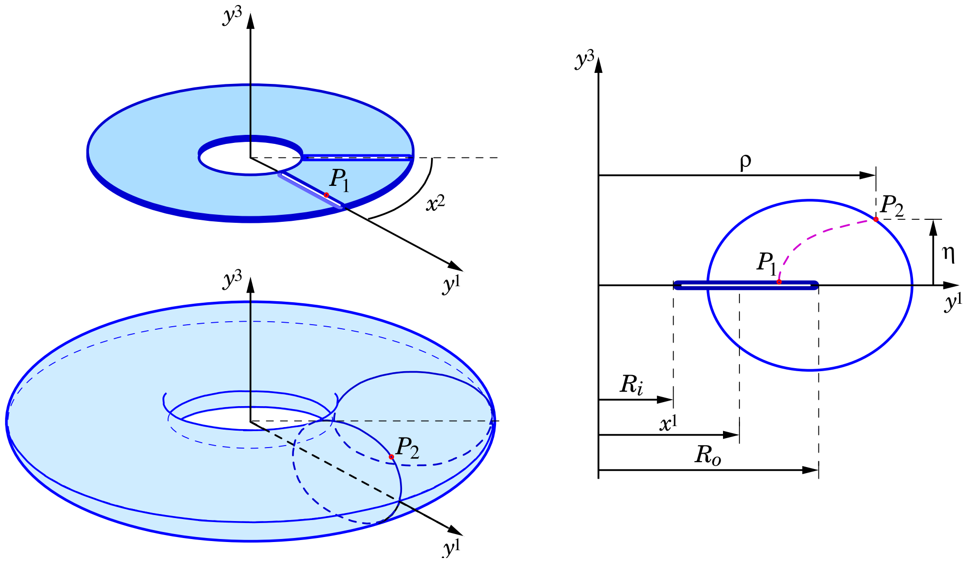

Fig. 1. Undeformed and deformed configurations of a flat toroidal membrane undergoing symmetric inflation.

gridding may lead to spurious symmetry breaking. Hence, an analytical approach is preferable in this analysis. In this work, we propose a perturbative symmetry breaking analysis of a toroidal membrane during inflation.

The appearance of asymmetry during inflation may be viewed as a bifurcation of the solution and subsequent loss of stability of the symmetric branch. Bifurcation modes with local necking behavior for an axisymmetrically inflated spherical membrane subjected to a prescribed increase in enclosed volume were studied by Needleman (1976). The occurrence of tensile instability and appearance of non-spherical modes during inflation of spherical shaped balloons has been discussed and demonstrated experimentally by Alexander (1991). In a more simple geometry of that of a flat circular membrane, finite inflation was found to lead to symmetry breaking instability through a supercritical pitchfork bifurcation Chaudhuri and DasGupta (2014). Stability investigations for thin membranes from the viewpoint of symmetry have been discussed in Eriksson and Nordmark (2016). A finite element formulation to capture the local buckling of cylindrical membrane tube under inflation and initial stress has been presented in Pamplona et al. (2006). Non-axisymmetric bulging and snap-through instability of short length tubular elastomeric balloons has been analyzed experimentally in Mao et al. (2014).

The symmetric inflation problem of toroidal membranes has been addressed previously in a number of studies (Hill, 1980; Kydoniefs, 1965; Kydoniefs and Spencer, 1965; Kydoniefs and Spencer, 1967; Li and Steigmann, 1995; Tamadapu and DasGupta, 2013). Membranes of toroidal topology, obtained by inflating a flat torus (both intrinsically and extrinsically) in undeformed state has been proposed and analyzed by Roychowdhury and DasGupta (2015a). In the present work, we study the geometric stability of such an axisymmetric flat toroidal membrane under finite inflation. We assume that the contribution of the asymmetric solution, if it appears, is small compared to the characteristic scale of the symmetric inflated shape. Using isochoric perturbative expansion, we search for asymmetric solution branches on the symmetric inflation path and study their stability. As the membrane is inflated, at a certain level of inflation, the symmetric solution branch undergoes spontaneous symmetry breaking through a supercritical pitchfork bifurcation. It is found that only the axisymmetry of the toroidal geometry is broken, while reflection symmetry of the torus about the equatorial plane is retained. Interestingly, the axisymmetry is restored at a higher level of inflation through a reverse subcritical pitchfork bifurcation. While a reverse subcritical pitchfork bifurcation is known to exist in dynamical systems Haberman (2001), its occurrence in structural stability problems is not commonly known. A two-member toggle truss also exhibits a symmetry breaking and restoration on its loading path Pecknold et al. (1985). However, while the symmetry is broken by a supercritical pitchfork bifurcation, the symmetry is restored by a reverse supercritical pitchfork bifurcation.

2. Symmetric inflation of a flat toroidal membrane

2.1. Membrane geometry

The undeformed geometry of the membrane consists of a stack of two identical flat annular membranes bonded along their inner (R_i) and outer (R_o) equators, as shown in Fig. 1. The axisymmetric finite inflation problem is formulated using a cylindrical coordinate system with x^1, x^2, and x^3 as, respectively, the radial, circumferential, and normal (along the thickness of the membrane) directions on the surface of the flat membrane. The mid-surface of the membrane is denoted by $x^3 = 0$. The reflection symmetry of the torus is preserved during the inflation process about a plane containing the inner and the outer equators. The deformed (undeformed) thickness of the membrane is taken as H (h). Considering symmetric inflation,