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Platonic crystal with low-frequency locally-resonant spiral structures: wave trapping, transmission amplification, shielding and edge waves

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ABSTRACT

We propose a new type of platonic crystal, which includes spiral resonators with low-frequency resonant vibrations. The special dynamic effects of the resonators are highlighted by a comparative analysis of dispersion properties of homogeneous and perforated plates. Analytical and numerical estimates of classes of standing waves are given and the analysis of a macrocell shows the possibility to obtain localization, wave trapping and edge waves. Applications include transmission amplification within two plates separated by a small ligament and a novel design to suppress low-frequency flexural vibrations in an elongated plate implementing a bypass system that re-routes waves within the mechanical system. Finally, we show the possibility to obtain Konenkov-Bloch edge waves for general boundary conditions.

1. Introduction

Metamaterials are microstructured media engineered to have properties that are not found in nature. The first models were developed in electromagnetism and optics and then extended to acoustics and elasticity (Cai and Shalaev, 2009; Capolino, 2009; Craster and Guenneau, 2012; Engheta and Ziolkowski, 2006). More recently, systems such as those described by the Kirchhoff-Love plate equations for flexural waves, labelled as *platonics* by McPhedran et al. (2009), have been studied. These flexural systems may show many of the typical anisotropic effects of photonics such as ultra-refraction, negative refraction and Dirac-like cones (Antonakakis et al., 2014a; Farhat et al., 2010; McPhedran et al., 2015; Smith et al., 2012; Torrent et al., 2013). Structured plates may be also employed as cloaking devices (Farhat et al., 2009a; 2009b; Misseroni et al., 2016; Stenger et al., 2012) as a result of inhomogeneous and anisotropic constitutive properties and axial prestress (Brun et al., 2014; Colquitt et al., 2014; Jones et al., 2015).

One of the main properties of the biharmonic equation of motion governing the propagation of flexural waves is the decomposition into Helmholtz and modified Helmholtz equations, associated with the presence of propagating and evanescent waves, respectively. Such waves can be coupled via the boundary or interface conditions. In most configurations, flexural waves are dominated by their Helmholtz component (McPhedran et al., 2009) and the homogenized equation can be of

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parabolic type at special frequencies (Antonakakis et al., 2014a; McPhedran et al., 2015). However, short range wave scattering and Bragg resonance can be strongly influenced by the evanescent waves.

Periodic structures play a major role in this field (Brillouin, 1946), since they create band gaps. These are frequency ranges where waves cannot propagate through the periodic system, leading to possible applications as acoustic and mechanical wave filters, vibration isolators and seismic shields. Partial band gaps can lead to anisotropic wave response that can be used to obtain focusing and localization (Bigoni et al., 2013; Brun et al., 2010; Piccolroaz et al., 2017) as well as polarization properties (Lai et al., 2011; Ma et al., 2016).

Two physical mechanisms can open band gaps: Bragg scattering and local resonance (Hussein et al., 2014; Mead, 1996). Bragg scattering is associated with the generation of band gaps at wavelengths of the same order as the unit cell around frequencies governed by the Bragg condition $a = n(\lambda/2)$ $(n = 1, 2, 3, \cdots)$, where a is the lattice constant of the periodic system and λ is the wavelength (Movchan et al., 2009). Local resonances correspond to internal resonances due to the microstructure and they can be obtained from array of resonators, as suggested in the seminal work (Liu et al., 2000). Local resonances open tiny band gaps that can be at low frequencies (Haslinger et al., 2017; Xiao et al., 2012a; 2013). Inertial amplification mechanisms that can widen stop band intervals have been proposed in Koç et al. (2005), Acar and Yilmaz (2013) and Frandsen et al. (2016).

As an introductory example, the numerical experiments reported in Fig. 1 show the capacity of the proposed metamaterial to generate localized flexural wave propagation along designed patterns.

Systems of internal resonators can be implemented to design a lightweight “wave bypass” structure that would divert the vibrations away from load-bearing elements and then be damped out easily. The bypass structure is essentially a highly directive system that re–routes the waves around the main structure, which is then shielded from vibrations within the unwanted frequency range (Brun et al., 2013a; 2013b; Carta et al., 2017b). The general theory for the design and the study of bypass structures in the case of monodimensional flexural systems has been detailed in Carta and Brun (2015) and Carta et al. (2016) and has been applied to study the dynamic behavior of a real bridge (Carta et al., 2017a). The design principles take inspiration from waveguides used in many areas of science and engineering to funnel or direct waves along particular paths (Timoshenko, 1980) and are here applied for the first time to plate structures.

We show that edge waves can propagate in the proposed microstructured plate for general boundary conditions. Konenkov was the first to show the existence of flexural waves that propagate along the edge and exponentially decay in amplitude in the direction perpendicular to the boundary of homogeneous Kirchhoff–Love plates (Konenkov, 1960). A necessary condition for the existence of Konenkov waves in homogeneous plates is the presence of Neumann free boundary conditions.

A periodicity in the geometry of the edge or a periodic diffraction grating embedded in a medium can lead to a class of edge waves (Popov, 2012). They have been called Rayleigh–Bloch waves since they combine the amplitude decay in the direction perpendicular to the edge with the periodicity in the edge tangential direction. They have been referred to with different names in water waves (Evans and Linton, 1993), plasmonics (Pendry et al., 2004), antenna theory (Sengupta, 1959), scalar systems (Wilcox, 1984), vector elasticity (Colquitt et al., 2015) and Kirchoff–Love plates (Evans and Porter, 2008; Haslinger et al., 2016), where they are created by a periodic distribution of pinned points . In the case of flexural waves, we use the definition of Konenkov–Bloch waves.

The platonic crystal proposed in this paper can be used as an efficient mechanism for multi-purpose applications. First of all, due to its internal microstructure, it has the capacity of showing several low-frequency resonance modes in the acoustic branch, thus improving the results presented in Bigoni et al. (2013) for the case of vector elasticity and in Haslinger et al. (2017) for plates. Furthermore, transmission may be enhanced in correspondence of the resonance modes. In this sense, the proposed system can represent a valid alternative to plates with rigid pins (Haslinger et al., 2012) or with surface corrugations (Hao et al., 2011). The platonic crystal introduced in this paper can also be employed to divert waves away from a structure that needs to be protected, with the same spirit as the bypass systems developed in Brun et al. (2013a,b) and Carta et al. (2017b) for bridges and tall building and in Colombi et al. (2017) for seismic waves in half-spaces, but applied to plates in the present case. Finally, the proposed periodic system can support propagating edge waves for any type of classical boundary conditions, while homogeneous plates admit the propagation of Konenkov–Bloch waves only in the case of a free edge (Evans and Porter, 2008; Norris et al., 2000).

The present paper is organized as follows. In Section 2 we present the governing equations and the geometry of the microstructure. Dispersion properties are obtained and discussed in Section 3, while asymptotic estimates of the first internal resonance frequencies and modes are presented in Section 4. In Section 5 we show different applications which include wave localization, transmission amplification, a bypass system for flexural waves and the existence of Konenkov-Bloch waves for all classical boundary conditions. Final considerations in Section 6 conclude the paper.

2. The platonic system of spiral resonators

We consider flexural vibrations in Kirchhoff–Love plates. In the time-harmonic regime, the transverse displacement $W(\mathbf{x})$ satisfies the fourth-order biharmonic equation

$$\nabla^4 W(\mathbf{x}) - \beta^4 W(\mathbf{x}) = 0, \quad \beta^4 = \frac{\rho h}{D}\omega^2. \tag{1}$$