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Research paper

A revised time-varying mesh stiffness model of spur gear pairs with tooth modifications

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ABSTRACT

Based on the thin slice assumption, a revised time-varying mesh stiffness (TVMS) model of spur gear pairs with tooth modifications is developed. The spur gear is divided into many individual slices along tooth width, and considering the revised fillet-foundation stiffness, the nonlinear contact stiffness, the extended tooth contact and the tooth profile errors, the stiffness of each slice gear pair is figured out. According to the relationship between the deformation and the total stiffness in mesh period, the TVMS of spur gear pairs can be worked out. Meanwhile, relative to the finite element (FE) method, the errors of the proposed method under different modification quantities are discussed. The proposed method is more accurate than those previous methods but there are still some errors. Taking the FE model as a benchmark, the TVMS is further revised based on a simple model updating technique. Based on the revised model, the effects of the tooth width and torque on mesh stiffness are also studied. The result shows that based on the proposed method, the TVMS under any given modification quantities in a suitable range can be calculated accurately.

1. Introduction

Tooth modification is an important method to reduce vibration and noise, which can be carried out by tooth profile and lead crowning modifications. In recent years, many scholars have studied the vibration and noise behaviors of gears with tooth modifications [1–9]. Lin and He [1] developed a finite element (FE) method to calculate the transmission error of gear transmission systems with machining errors, assembly errors and modifications. Wang et al. [2] and Maatar and Velex [3] analyzed the effects of gear modifications on the contact and dynamic characteristics of a gear pair. Bruyère et al. [4] investigated the transmission errors of modified gears and obtained the optimal profile modifications which can minimize the transmission error fluctuations. Velex et al. [5] proposed a design criterion for tooth modifications minimizing dynamic tooth loads. They also pointed out that the transmission error fluctuations are related to the tooth modifications [6]. Bahk and Parker [7] studied the effects of tooth profile modifications on the vibration of spur planetary gears. Ma et al.

Abbreviations: 3D, Three-dimensional; CW1∼CW9, Nine modification quantities for the gear pair 1; CL1∼CL8, Eight modification quantities for the gear pair 2; FE, Finite element; FEM, Finite element model; PAM, Proposed analytical model; TVMS, Time-varying mesh stiffness.

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Nomenclature

C_q	Lead crowning quantity
C	Tooth profile error in mesh position caused by tip relief along horizontal direction (x-direction in Fig. 2b)
C_a	Amount of profile modification
dz	Width of each piece spur gear pair
E	Young's modulus
E_{pc}	Tooth errors of each slice gear caused by lead crowning relief along horizontal direction (x-direction in Fig. 2b)
E_{p}^{i}	Total tooth profile error of the ith tooth pair of each slice gear pair along line of action
E_{r}^{i}	Static transmission error of the ith tooth pair of each slice gear pair at meshing point
F	Total meshing force
F_i ($i=1, 2$)	Meshing force of ith meshing tooth pair
F_n ($n=1 \sim N$)	Total meshing force of the nth piece tooth
j	Number of meshing position
k_{a}, k_{b}, k_{s}	Axial compressive stiffness, bending stiffness and shear stiffness
k_n	TVMS of the nth piece tooth
k_{tooth}	Total mesh stiffness of meshing teeth pairs
k_{tooth}^{i}	Stiffness of the ith tooth pair
k_{h}^{*}	Hertzian contact stiffness
k_{h}^{i}	Nonlinear Hertzian contact stiffness of the ith tooth pair
k_{mean}	Mean mesh stiffness of gear pairs
k_{t1}^{i}, k_{t2}^{i}	Tooth stiffness of the ith tooth pair and subscripts 1 and 2 denote the driving and driven gears, resepectively.
k_{f}	Stiffness of fillet-foundation
K	Total mesh stiffness of spur gear pairs with tooth modifications
L	Width of the tooth
L_a	Length of profile modification
lsf_i	Load-sharing ratio of the ith tooth pair
q	Number of tooth pair
N	Number of slice gear pair
R	Radius of the lead crowning circular curve
T_l	Torque applied to the driving gear
Z	Number of tooth
z_n	Coordinate of each sliced spur gear pair along the axial direction (z-direction in Fig. 3c)

Greek symbols

λ_1, λ_2 ($i=1, 2$)	Coefficients of the fillet-foundation stiffness, subscripts 1 and 2 denote the driving gear and driven gear, respectively
λ_k	TVMS correction coefficient
ν	Poisson's ratio

[8] proposed a mesh stiffness model for profile shifted gears with addendum modifications and tooth profile modifications, and determined the optimum profile modification curve under different amounts of tooth profile modifications.

The vibration characteristics of gear transmission systems will be significantly affected by the time-varying mesh stiffness (TVMS) [10–13], which will change with tooth modifications. In the earlier study, the TVMS of healthy spur gear pairs can be evaluated by analytical method on the basis of the potential energy method in elastic mechanics [14–18]. However, the stiffness of the spur gear with tooth modifications or tooth faults is difficult to be determined accurately by this analytical method, and the effects of gear errors are usually ignored. Subsequently, based on the analytical method, Chen and Shao [19] took the gear errors into account and developed a TVMS calculation model, in which the relationship between the mesh deformation and the total mesh stiffness in mesh period is determined. Furthermore, they also proposed a more general calculation method for both healthy and tooth root crack cases, and developed an analytical model to calculate the mesh stiffness of spur gear pairs with non-uniformly distributed tooth root crack, and the dynamic simulation of spur gears with tooth root crack propagating along tooth width and crack depth is also carried out [20–22]. The effects of extended tooth contact (ETC) on TVMS should not be ignored in TVMS calculation [23,24] and Ma et al. [25–28] established an improved TVMS model for healthy gear pairs, cracked spur gears and gear pairs with tip relief, in which the transition curve, revised fillet-foundation stiffness, nonlinear contact stiffness and ETC were considered. Liu et al. [29] studied the mesh stiffness of a gear pair with tooth profile modification by analytical method and determined the optimal modification amount.