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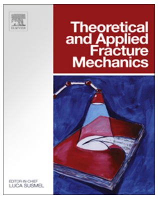

Fatigue life assessment of notched round bars under multiaxial loading based on the total strain energy density approach

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ABSTRACT

The main purpose of this paper is the fatigue assessment in lateral U-shaped notched round bars under bending-torsion loading. Despite its importance in the context of mechanical design, very little work has been done in this field. The fatigue life prediction model relies on the assumption that both the smooth and the notched samples fail when a critical value of the total strain energy density is reached. The *modus operandi*, in a first instance, consists of developing a fatigue master curve that relates the total strain energy density and the number of cycles to failure using smooth specimens subjected to strain-controlled conditions. In a second stage, the total strain energy density of the notched samples is computed from representative hysteresis loops obtained through a three-step procedure: reduction of the multiaxial stress state to an equivalent stress state using a linear-elastic finite-element model; definition of an effective stress range on the basis of the Theory of Critical Distances; and generation of a hysteresis loop applying the Equivalent Strain Energy Density concept in conjunction with the calculated effective stress range. The comparison between the experimental and the predicted lives has shown a very good correlation, with all points within a factor of 2.

1. Introduction

Most mechanical components with circular cross-sections contain notches because of design requirements. When subjected to multiaxial loading histories, the stress-strain responses at the geometric discontinuities may result in complex fatigue problems, even in cases of low plastic deformation. In this context, the full understanding of the notch effect is pivotal to develop safe and durable products. Moreover, due to the increasingly short product life cycles, lower batch sizes, and the growing need to reduce overall costs, products are developed in smaller time frames. Thus, the rapid assessment of fatigue life in notched members subjected to non-trivial loading scenarios is indispensable to increase efficiency and, ultimately, attain engineering excellence.

Fatigue life prediction models based on local approaches require detailed information about the stress-strain state at the notch root [1–6]. One of the most popular methods to deal with notch fatigue problems was formulated by Neuber [1], who stated that the geometric mean value of both the stress and strain concentration factors is constant at any load state, and equals the elastic stress concentration factor. Nevertheless, despite its popularity, strains at the notch root tend to be over-estimated [7]. Other popular methods are those based on the strain energy density [8]. Molski and Glinka [3] proposed the Equivalent Strain Energy Density (ESED) concept, which assumes that the strain energy density of the material in the yielded zone is virtually the same as the strain energy density assuming the material to be entirely elastic. Although in certain circumstances, it is more accurate than the above-mentioned method, notch root strains tend to be underestimated, when nominal stresses approach the yield stress [9]. A more general formulation, based on a fatigue master curve evaluated from the sum of the positive elastic and plastic strain energy densities of representative cyclic hysteresis loops, was suggested by Ellyin et al. [10,11]. Lazzarin et al. [6,12] developed a volume-based approach, in which the SED calculations are carried out in a material-related control volume. A recent literature review on strain energy density approaches can be found in Ref. [13].

The Theory of Critical Distances (TCD) is another successful group of methods capable of accounting for the notch effect on fatigue problems [14,15]. The different methods have in common the fact that the effective stresses at the fatigue process zone are defined on the basis of a characteristic material length, i.e. the well-known critical distance. The origin of this theory, introduced by Neuber, date back the middle of the last century [16]. In essence, the so-called Line Method (LM) states that the reference

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stress for fatigue assessment can be obtained by averaging the linear-elastic stress profile over a straight line emanating from the notch root. Some years later, Peterson [17] suggested that the reference stress could be computed from the linear-elastic stress profile at a given distance from the notch root, considerably simplifying the problem. This approach is known as the Point method (PM). An overview on different applications of the TCD to fatigue problems can be found elsewhere [18].

The benefit of assessing fatigue lives using local elasto-plastic stresses and strains from pseudo-elastic stresses in a practical perspective, i.e. simplicity, computational overhead, or overall costs, has encouraged new research on notch fatigue correction [19,20]. However, the above-mentioned approaches have some limitations. On the other hand, although fatigue life prediction models for uniaxial conditions are sufficiently mature, the same cannot be said for multiaxial loading. Furthermore, the identification of a universally accepted fatigue damage parameter has not been yet achieved [21,22]. Therefore, the development of reliable multiaxial fatigue life methodologies for notched components remains a challenging problem, and requires further research.

The present paper deals with the fatigue life prediction of notched round bars with lateral notches undergoing in-phase bending-torsion loading. Despite the relevance of lateral notched round bars in the context of mechanical design, few studies have been conducted so far [23–33]. It should likewise be noted that the existing research has been mainly focused on transverse circular holes, or circumferential notches. Lateral notches in round bars subjected to bending-torsion histories have not been sufficiently explored. The purpose of this study is to investigate this issue in greater depth. The paper starts with the description of the multiaxial fatigue life prediction model. Section 3 addresses the material employed, the low-cycle fatigue tests conducted to obtain the fatigue master curve, and the multiaxial fatigue test program of the notched specimens; as well as the linear-elastic finite-element model developed to compute the stress state at the notch. Section 4 analyses the total strain energy density of the smooth specimens; the loading effect on fatigue behaviour in the notched specimens; and ends with the comparison of the experimental and predicted fatigue lives. The last section presents the concluding remarks.

2. Fatigue life prediction model

The main steps of the fatigue life prediction model are schematised in Fig. 1. Basically, it assumes that both smooth and notched samples accumulate the same damage and have the same lives if the stress-strain histories at the initiation sites are identical; and that fatigue failure occurs when the total strain energy density defined as the sum of both the plastic and the positive elastic components at the initiation sites reaches a critical value.

The first step is devoted to the analysis of the stress-strain response of the material from smooth specimens under fully-reversed strain-controlled conditions. For each test, a hysteresis loop is selected, and the total strain energy density is evaluated. The information collected for various strain amplitudes enables the definition of a fatigue master curve in the form:

$$\Delta W_T = \kappa t (2N_f)^{\alpha t} + \Delta W_{0t} \tag{1}$$

where ΔW_T is the total strain energy density, κt and αt are constants, N_f is the number of cycles to failure, and ΔW_{0t} is the tensile elastic energy at the material fatigue limit. The fatigue master curved obtained in this step is represented in Fig. 1(d). The use of the positive elastic strain energy density makes this parameter sensitive to the mean stress effect [10,11].

With regard to the notched samples, the multiaxial stress states at the notch caused by different combinations of normal and shear stresses (see Fig. 1(a)) are reduced to uniaxial stress states through the computation of the von Mises equivalent stress range (Fig. 1 (b)). Then, this equivalent uniaxial stress state is averaged using the Line Method (LM) of the Theory of Critical Distances (TCD). The critical distance ($D_{LM} = 2a_0$) is defined from the El Haddad [34] parameter

$$a_0 = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2 \tag{2}$$

where ΔK_{th} is the range of the threshold value of the stress intensity factor, and $\Delta\sigma_0$ is the fatigue limit of the unnotched specimen. Such constants are evaluated at the same stress ratio of the notched component to be assessed. After that, using the averaged von Mises stress range and the Equivalent Strain Energy Density concept [35], a representative hysteresis loop is generated (Fig. 1(c)). Finally, the total strain energy density of the hysteresis loop is inserted into the fatigue master curve (Eq. (1)) to estimate the fatigue life (Fig. 1(d)).

3. Experimental and numerical procedure

3.1. Material

This study was conducted using a DIN 34CrNiMo6 high strength steel, oil quenched and tempered (Q&T), supplied in the form of 20 mm-diameter bars. The production process comprised an autenitisation at 850–880 °C for approximately 30 min, followed by oil cooling and temper at about 660 °C for at least 2 h, and air cooling. Its main mechanical properties are summarised in Table 1.

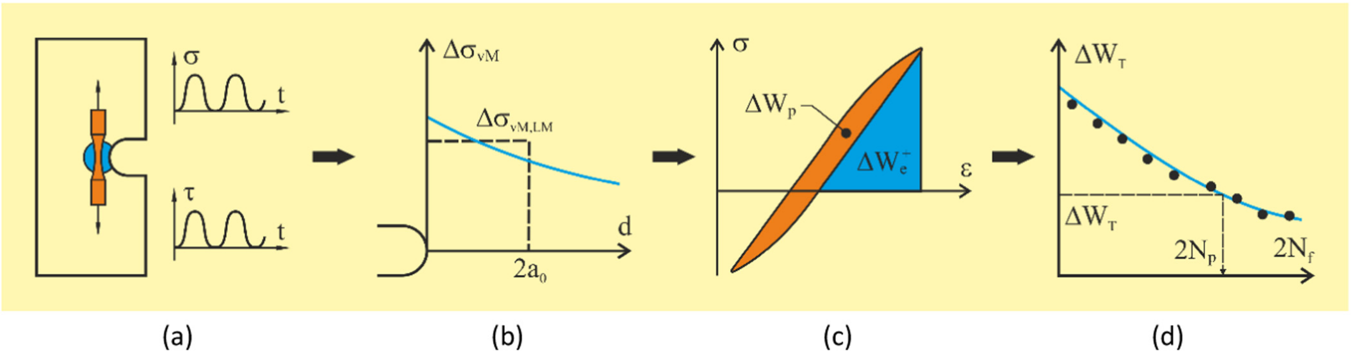

Fig. 1. Fatigue life prediction approach based on the total strain energy density evaluated at the initiation sites from hysteresis loops obtained through the ESED concept and an average stress given by the LM of the TCD: (a) reduction of the multiaxial stress state to an equivalent uniaxial stress state; (b) computation of the effective stress at the fatigue process zone; (c) calculation of the total strain energy density; (d) lifetime assessment using the fatigue master curve.

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