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Energy-based approach for fracture assessment of several rocks containing U-shaped notches through the application of the SED criterion

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ABSTRACT

This work presents an energetic continuum approach for the fracture assessment of rocks containing U-shaped notches and subjected to Mode I loading conditions. Three different methodologies are proposed, all based on the premise that brittle failure will occur when the average strain energy density over a certain control area reaches a critical value that only depends on the material, as stated by the Strain Energy Density (SED) criterion.

The first method proposed (A) deals with the application of the SED criterion through an expression with a series of already tabulated parameters, which are particularised for the analysed rocks by rational extrapolation. By contrast, the second method (B) aims to obtain numerically the previously extrapolated parameters, and the third method (C) directly relates the strain energy density with the applied load, without the use of those parameters.

The research is based on the results obtained from an exhaustive experimental programme comprising 300 fracture specimens tested in four-point bending conditions. These tests combine parallelepiped samples made of six different types of rocks (two marbles, two limestones, a sandstone and a granite) and containing eight different notch radii (varying from 0.15 mm up to 15 mm).

Thus, this work aims to show the potential, capacity and limitations of the SED criterion in rock fracture analyses, by comparing the experimentally obtained fracture loads to those predicted by the three proposed methodologies.

1. Introduction

The brittle condition of intact rocks, their high inherent strength and low toughness make them very sensitive to the presence of any defect like discontinuities, microcracks, pores, grain boundaries, etc. However, notch-type defects generate smaller stress fields and usually develop larger fracture resistance than sharp cracks. Therefore, the two types of defects should not be treated in the same way. Besides, rocks are usually subjected to relatively high loads both in nature and in industrial exploitations, which makes them even more sensitive to stress risers. All this leads to the necessity of developing accurate tools for rock fracture assessment different from the over-conservative ones derived from linear elastic fracture mechanics, where sharp crack behaviour is usually assumed for many practical situations, even for those with notch-type defects. Numerous examples can be found of traditional and conservative rock fracture mechanics, where crack-type defects are assumed for different applications of rock cutting, hydraulic fracturing or underground excavation.[1–3]

Dealing with fracture analyses of notched components subjected to Mode I loading, different criteria have been proposed in the last few decades. However, establishing a general classification of these existing criteria is not easy, since the boundary between them may sometimes be diffuse. First, the Global Criterion should be mentioned, which states that fracture will occur when the notch stress intensity factor (NSIF) reaches a critical value, K_ρ^c, that depends on the material and notch radius.[4,5] On the other hand, Taylor[6] collects the Critical Distance methodologies, all of them using a characteristic material length parameter (the critical distance, L) when performing fracture assessment. Proceeding along similar tracks, Gómez et al.[7] presented a criterion for brittle or quasi-brittle materials under monotonic loading, based on the cohesive zone models proposed in the past by Dugdale and Barenblatt[8,9] to describe stress fields and fracture processes near the defect tip. This criterion has led to successful predictions of the fracture loads of notched specimens[10–13] in parallel to other methods based on

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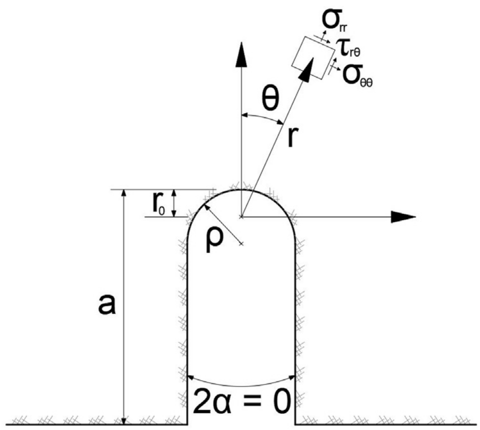

Fig. 1. Coordinate system and notch geometry with the corresponding nomenclature.

the Strain Energy Density (SED) concept.[14–16] The SED criterion combines the so-called elementary volume proposed by Neuber[17] and the local Mode I concept first proposed by Erdogan and Sih.[18] It claims that brittle failure will occur when the average strain energy density over a given, well-defined, control volume is equal to a certain critical value (W_c) dependent on the material. This energetic approach has been successfully applied to assess both the static fracture[19–22] and the fatigue behaviour[23,24] of notched components (including welded structures) subjected to predominant Mode I or even Mixed Mode loading.[14–16,25] Here, blunt U-shaped notches (opening angle $2\alpha = 0$, see Fig. 1) will be studied in detail.

The application of the SED criterion in materials like steels or polymers has been widely studied by many authors.[10–16,26] However, despite its huge potential and advantages, its applicability in more heterogeneous and brittle materials like rocks requires further research, since it has not been until the last few years that it has begun to be applied to rocks, as for example by Aliha et al. or Berto et al.[27,28]. For this reason, trying to fill this knowledge gap, this paper focuses on the use of the SED criterion for the fracture prediction of six different rocks containing U-shaped notches and with notch radii varying from 0.15 mm to 15 mm. To this end, the study is based on the results of a systematic laboratory campaign performed by the authors of this article, [29,30] which successfully applied the Theory of The Critical Distances for the fracture assessment of those six rocks.

2. Strain Energy Density Criterion

This section gathers some theoretical background on the SED criterion, which has traditionally been used to express failure conditions for materials presenting both ductile and brittle behaviour.[26] According to the SED approach, the average strain energy density ($\bar{W}$) over a certain control volume is limited by a critical value (W_c). Thus, fracture will occur when:

$$\bar{W} = W_c \tag{1}$$

The critical strain energy W_c only depends on the material and its value is defined by the area under the corresponding stress-strain curve (Fig. 2).

In the case of elastic and quasi-brittle materials like rocks (Fig. 2a), W_c has a very simple expression and can be easily assessed through the conventional ultimate tensile strength (σ_u) and the Young's modulus (E):

$$W_c = \frac{\sigma_u^2}{2E} \tag{2}$$

On the other hand, the control volume over which the energy is averaged becomes an area (Ω) in plane stress or plane strain conditions. This area is defined by a radius R_c (see Fig. 3), which depends on the ultimate tensile strength σ_u, the fracture toughness K_{IC} and the Poisson's ratio ν of the material in the case of static loads. Yosibash et al.[31] provided an expression for R_c, which, in the case of $2\alpha = 0$, is as follows:

$$R_c = \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{K_{IC}}{\sigma_u}\right)^2 \tag{3}$$

Considering the polar coordinates represented in Fig. 1 and their 3D extension to cylindrical coordinates (r, θ, z), the strain energy at a certain point for an isotropic material obeying a linear elastic law is:

$$W(r,\theta,z) = \frac{1}{2E}\{\sigma_{\theta\theta}^2 + \sigma_{rr}^2 + \sigma_{zz}^2 + 2\tau_{r\theta}^2 - 2\nu(\sigma_{\theta\theta}\sigma_{rr} + \sigma_{\theta\theta}\sigma_{zz} + \sigma_{rr}\sigma_{zz} - \tau_{r\theta}^2)\} \tag{4}$$

The average strain energy density over the control area Ω depicted in Fig. 3 and defined by Eq. (3) can be expressed as:

$$\bar{W} = \frac{\int_\Omega W d\Omega}{\Omega} = \frac{1}{\Omega}\int_{-\bar{\theta}}^{\bar{\theta}} d\theta \int_{R_1(\theta)}^{R_2} W(r,\theta) r dr \tag{5}$$

These expressions are further developed for the general case of notches with different opening angles.[15,26,32] From the mathematical development of Eq. (5), Eq. (6) is derived for the calculation of the mean value of the SED, which simplifies to a great extent the analysis of fracture processes in notched components:

$$\bar{W} = 0.785 \cdot H\left(\nu, \frac{R_c}{\rho}\right) \cdot \frac{\sigma_{max}^2}{E} \tag{6}$$

This expression is valid for the particular case of U-shaped notches subjected to Mode I loading conditions, where σ_{max} is the maximum stress at the notch tip for the applied load and E is the Young's modulus of the analysed material. The function H, on the other hand, depends on the ratio R_c/ρ and on the Poisson's ratio ν for U-shaped notches, and may be tabulated as in 15,26.

Thus, rock fracture analyses can be directly made comparing the critical value of the strain energy (W_c) defined in Eq. (2) with the mean SED value ($\bar{W}$) provided by Eq. (6). With all this, only some basic mechanical properties of the material (E, ν, σ_u and K_{IC}) and the maximum principal stress at the notch tip (σ_{max}) are needed for fracture assessment, apart from the geometrical aspects of the notch.

3. Experimental programme

3.1. Materials

The application of the SED criterion has been particularised to six representative rocks with different properties and microstructures, in an attempt to cover a wide casuistic of samples and thus prove its validity. The chosen rocks are a biotite granite **(G)**, an oolitic limestone **(O)**, a Moleano limestone **(C)**, a Floresta sandstone **(F)**, a Carrara (Italian) marble **(I)** and a Macael marble **(M)**, all of them being isotropic and relatively homogeneous at the macro-scale. These rocks were previously analysed by the authors,[29,30] who performed the rigorous laboratory campaign that is taken as a basis in this article. Table 1 summarises some of the most relevant technical properties of each rock. More details on their microstructure and components can be consulted in 29,30.

3.2. Simple compression tests and Brazilian tests

60 compression tests were performed in total following UNE-EN 1926 and UNE-EN 14580[33,34] six in the case of the granite and oolitic limestone and twelve for the rest of the rocks. The deformations in the two main perpendicular directions were measured using strain gauges, which allowed the Young's modulus and the Poisson's ratio to be