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Multistability and instability of competitive neural networks with non-monotonic piecewise linear activation functions

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A B S T R A C T

This paper addresses the issue of multistability for competitive neural networks. First, a general class of continuous non-monotonic piecewise linear activation functions is introduced. Then, based on the fixed point theorem, the contraction mapping theorem and the eigenvalue properties of strict diagonal dominance matrix, it is shown that under some conditions, such n-neuron competitive neural networks have exactly 5^n equilibrium points, among which 3^n equilibrium points are locally exponentially stable and the others are unstable. Moreover, it is revealed that the neural networks with non-monotonic piecewise linear activation functions introduced in this paper can have greater storage capacity than the ones with Mexican-hat-type activation function and nondecreasing saturated activation function. In addition, unlike most existing multistability results of neural networks with nondecreasing activation functions, the location of those obtained 3^n locally stable equilibrium points in this paper is more flexible. Finally, a numerical example is provided to illustrate and validate the theoretical findings via comprehensive computer simulations.

1. Introduction

In the past decades, neural networks have been extensively studied due to their applications in image processing, pattern recognition, associative memories and so on [1–3]. In these applications, it is essential that neural networks involved are multistable, that is, they exhibit a large number of locally stable equilibrium points. For example, in the application of associative memory, the addressable memories are usually stored as stable equilibrium points and the process of memory attainment is that the network converges to a certain stable equilibrium point. The number of locally stable equilibrium points corresponds to the storage capacity of neural networks. Thus, it is more desirable that neural networks can have multiple locally stable equilibrium points. Design of such practical neural networks mandates the necessity for dynamical analysis

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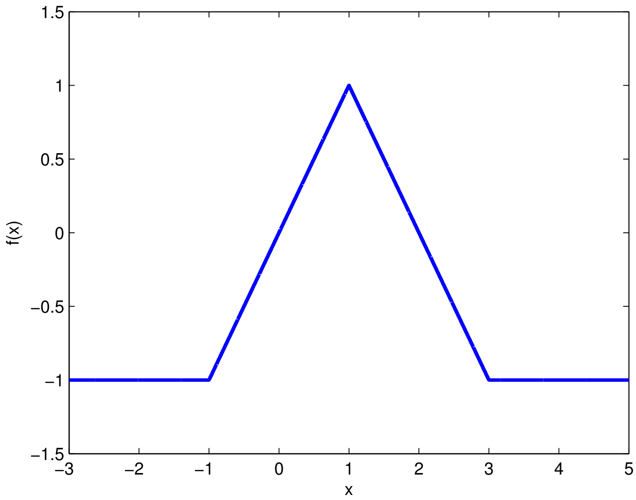

Fig. 1. Mexican-hat-type activation function (1).

of multiple equilibrium points. So far, some valuable results for multistability of neural networks have been reported (see, e.g. [4–18] and references therein).

It has been well recognized that the multistability analysis of neural networks critically depends upon the type of activation functions used. In the above-mentioned works as well as most existing literature on multistability, the frequently used activation functions are primarily sigmoidal activation functions [5,7,8,15,18], nondecreasing saturated activation functions [4,5,7,11,15], and piecewise linear activation functions [6,9,12–14,17], which share the common feature that they are all monotonically increasing. Recent paper [19] introduced a class of non-monotonic piecewise linear activation function which is called Mexican-hat-type activation function defined by (see Fig. 1)

$$f(x) = \begin{cases} -1, & -\infty < x < -1, \\ x, & -1 \le x \le 1, \\ -x+2, & 1 < x \le 3, \\ -1, & 3 < x < +\infty. \end{cases} \tag{1}$$

Under the assumption that the index set $\{1, 2, \dots, n\}$ can be decomposed into four subsets with respect to different external input ranges, the multistability of Hopfield neural networks was studied in [19], by tracking the dynamics of each state component. It was shown therein that under the imposed assumption, the n-neuron Hopfield neural networks have $3^{\sharp N_2}$ equilibrium points in all, $2^{\sharp N_2}$ of them are locally stable and the others are unstable, where $\sharp N_2$ denotes the number of elements in the second index subset N_2.

With the inspiration from Mexican-hat-type activation function (1) and the aim to increase the storage capacity of neural networks, in this paper, we consider another class of continuous non-monotonic piecewise linear activation functions [20,21], which have the following form (see Fig. 2):

$$f_i(x) = \begin{cases} u_i, & -\infty < x < p_i, \\ l_{i,1}\,x + c_{i,1}, & p_i \le x \le r_i, \\ l_{i,2}\,x + c_{i,2}, & r_i < x < q_i, \\ l_{i,3}\,x + c_{i,3}, & q_i \le x \le s_i, \\ v_i, & s_i < x < +\infty, \end{cases} \tag{2}$$

where $p_i,\ r_i,\ q_i,\ s_i,\ u_i,\ v_i,\ l_{i,1},\ l_{i,2},\ l_{i,3},\ c_{i,1},\ c_{i,2},\ c_{i,3}$ are constants with $-\infty < p_i < r_i < q_i < s_i < +\infty,\ l_{i,1} > 0,\ l_{i,2} < 0,\ l_{i,3} > 0,\ u_i = f_i(q_i)$, and $v_i > f_i(r_i),\ i = 1, 2, \dots, n$. It should be mentioned that the