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Effect of harvesting quota and protection zone in a nonlocal dispersal reaction–diffusion equation[☆]

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ABSTRACT

In this article, we study the nonlocal dispersal reaction–diffusion equation with spatially non-homogeneous harvesting

$$\begin{cases} u_t = \int_{R^N} J(x-y)u(y,t)dy - u(x,t) & \\ \quad + au(1-u) - ch(x)p(u), & in\ \Omega \times (0,\infty), \\ u(x,t) = 0 & in\ \mathbb{R}^N \setminus \Omega \times (0,\infty), \\ u(x,0) = u_0(x), & in\ \bar{\Omega}, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $a > 0$ and $c > 0$ are constants, J is a continuous and nonnegative dispersal kernel, $p(u)$ is a harvesting response function which satisfies Holling type II growth condition, and $h(x)$ is the harvesting distribution function which may be zero in some subdomain of Ω. We first establish the existence and uniqueness of positive stationary solutions. Then we show that when the intrinsic growth rate a is larger than the principal eigenvalue of the protection zone, then the population is always sustainable; while in the opposite case, there exists a maximum allowable catch to avoid the population extinction. The existence of an optimal harvesting pattern is also shown.

1. Introduction

In this paper we consider the nonlocal dispersal reaction–diffusion equation with spatially non-homogeneous harvesting

$$\begin{cases} u_t = \mathcal{D}u + au(1-u) - ch(x)p(u), & in\ \Omega \times (0,\infty), \\ u(x,t) = 0 & in\ \mathbb{R}^N \setminus \Omega \times (0,\infty), \\ u(x,0) = u_0(x), & in\ \Omega, \end{cases} \tag{1.1}$$

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where the function $u(x,t)$ represents the density of a species at location x and time t; the habitat of the species is a bounded smooth domain Ω of $\mathbb{R}^N$; the constant $a>0$ is the intrinsic growth rate. The harvesting effort is described by the term $ch(x)p(u)$ and $c>0$ is the harvesting rate, $p(u)$ is a harvesting response function which satisfies Holling type II growth condition, $h(x)$ is the harvesting distribution function which may be zero in some subdomain $\Omega_0 \subset \Omega$, i.e. Ω_0 is a protection zone of the species. The initial function $u_0(x) \in C(\bar{\Omega})$ is nonnegative and nontrivial, and

$$\mathcal{D}u(x,t) = \int_{R^N} J(x-y)u(y,t)dy - u(x,t) \tag{1.2}$$

represents the nonlocal dispersal operator with continuous and nonnegative dispersal kernel J. Throughout this paper, we make the following assumptions on J,p and h:

(J) The kernel J is assumed to be a $C(\mathbb{R}^N)$ function with a compact support. Moreover, $J(0)>0, J\geq 0$, $J(-x)=J(x)$ and $\int_{R^N} J(x)dx = 1$;

(p) $p \in C^1([0,+\infty)), p(0)=0, p'(u)>0$ for $u\in[0,\infty)$, and $\lim_{u\to\infty} p(u) = 1$; and

(h) $h \in L^\infty(\Omega), 0 \leq h(x) \leq M$ for $x \in \bar{\Omega}$ and some $M>0$, and $\int_\Omega h(x)dx = 1$, where M is the maximum harvesting density at any location x.

For simplicity, we assume that

$$p(u) = \frac{u}{b+u}.$$

For more detailed background of this model, the readers can refer to [1–4] and [5]. Set

$$\Omega_0 = \{x \in \Omega | h(x) = 0\},$$

and assume that $\Omega_0 \subset \Omega$ has a smooth boundary. Then Ω_0 can be looked as a protection zone or no-harvesting zone. Since $h(x)$ satisfies the condition (h), then we have

$$1 = \int_\Omega h(x)dx \leq M(|\Omega| - |\Omega_0|),$$

where $|\Omega|$ is the Lebesgue measure (area if in $\mathbb{R}^2$) of a region Ω.

A solution of (1.1) which is time independent is called a stationary solution. We are interested in the positive stationary solutions of (1.1) and so we consider the nonlocal equation

$$\begin{cases} \int_{R^N} J(x-y)u(y)dy - u(x) + au(1-u) - ch(x)p(u) = 0, & in\ \Omega, \\ u(x) = 0 & in\ \mathbb{R}^N \setminus \Omega. \end{cases} \tag{1.3}$$

By a solution to (1.3) we mean a function $u \in L^1(\mathbb{R}^N)$ which verifies (1.3) almost everywhere. If $u>0$ in $\bar{\Omega}$, we say it is a positive solution.

Note that the problem (1.1) can be viewed as the nonlocal dispersal counterparts of the following problem associated to random dispersal operator

$$\begin{cases} u_t = \Delta u + au(1-u) - ch(x)p(u), & in\ \Omega \times (0,\infty), \\ u(x,t) = 0, & on\ \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x), & in\ \Omega, \end{cases} \tag{1.4}$$

The readers can refer to [5] where the corresponding Neumann boundary value problem of (1.4) was investigated.

Let $\lambda_1^M(\Omega_0)$ be the principal eigenvalue of

$$\begin{cases} \Delta\varphi + \lambda\varphi = 0, & in\ \Omega_0, \\ \varphi = 0, & in\ \partial\Omega_0 \cap \Omega, \\ \dfrac{\partial\varphi}{\partial n} = 0, & in\ \partial\Omega_0 \cap \partial\Omega. \end{cases}$$