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Analytic study on optical solitons in parity-time-symmetric mixed linear and nonlinear modulation lattices with non-Kerr nonlinearities

Asad Zubair[a], Nauman Raza[a], Mohammad Mirzazadeh[b], Wenjun Liu[c,*], Qin Zhou[d,*]

[a] *Department of Mathematics, University of the Punjab, Quaid-e-Azam Campus, Lahore, Pakistan*
[b] *Department of Engineering Sciences, Faculty of Technology and Engineering, East of Guilan, University of Guilan, P.C. 44891-63157, Rudsar-Vajargah, Iran*
[c] *State Key Laboratory of Information Photonics and Optical Communications, and School of Science, P. O. Box 122, Beijing University of Posts and Telecommunications, Beijing 100876, People's Republic of China*
[d] *School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China*

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ABSTRACT

This paper studies soliton solutions in parity-time-symmetric mixed linear and nonlinear optical lattices with Kerr and non-Kerr law nonlinearities. Four different kinds of rough mediums namely, parabolic law, power law, dual power law and polynomial law are considered in the context of non-Kerr media. With the help of the inverse engineering scheme, different types of soliton solutions are presented. The extracted outcomes show that exact bright and dark soliton solutions can exist for those different physical states.

1. Introduction

Optical lattices have become a widely used and highly recognized tool to study the quantum physics of many bodies with special relevance for solid state type systems. Wave propagation in periodic lattices is familiar to display many fundamental properties that appear due to the existence of allowed bands and forbidden gaps. One of the most interesting outcomes of nonlinearity in such periodic systems is the presence of self-localized entities, which are best known as lattice solitons. This type of self-localized modes has currently engaged substantial attention in many branches of science such as Bose–Einstein condensates [1], biological physics [2], solid state physics [3] and nonlinear optics [4]. Nonlinear waveguide lattices [5] provides the productive territory for the direct experimental inspection and study of one-dimensional lattice solitons [6–10].

With the commencement of the notion of parity-time (PT)-symmetric complex-valued potential in quantum mechanics by Bender and Boettcher in 1998 [11], recently the research interests on optical lattice solitons have been shifted from the real domain to the complex one [12–23]. Optics has provided a complete workbench for the perception of PT potentials, which are simply equivalent to a complex refractive-index distribution $s(x) = s_R(x) + is_I(x)$, allowing exploratory inspections of new tropical outcomes both in the spatial and in the temporal domain [24–27]. Such a potential indicates an even symmetry for the real part of the complex potential and an odd profile for the imaginary part of the complex potential. PT symmetry has found applications in many areas spanning from microwave cavities [28] and electronics [29,30] to quantum field theory [31] and from PT-symmetric quantum oscillators [11] to linear [32] and nonlinear optics [33–35].

* Corresponding authors.
E-mail addresses: jungliu@bupt.edu.cn (W. Liu), qinzhou@whu.edu.cn (Q. Zhou).

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The study of nonlinear waves is modulated by different types of nonlinear Schrödinger equations (NLSEs) including the integral order NLSE equation, which can be used to describe the propagation of solitons through optical fibers for trans-oceanic and trans-continental distances [36–39]. The integral order NLSE equation is studied in many sections of engineering, theoretical physics and applied mathematics. In particular, it appears in the study of biochemistry, fluid dynamics, plasma physics and nonlinear optics.

The aim of the present work is to study the soliton solutions of an integral order NLSE equation regulated with PT-symmetric mixed linear and nonlinear optical lattices in Kerr law and non-Kerr law media. As a result, analytic optical solitons, along with the corresponding PT-symmetric mixed linear and nonlinear complex potentials are reported. The physical model studied in this work is as follows:

$$\mathrm{i}q_z^m + \frac{1}{2}q_{\mathrm{xx}}^m + P_l(x)q^m + P_{\mathrm{nl}}(x)|q|^2q^m + F(|q|^2)q^m = 0. \tag{1}$$

In Eq. (1), $q(x, z)$ represents the complex field amplitude, where z and x are the normalized longitudinal and transverse coordinates. Where m belongs to some integer value. The space-regulated parameters $P_l(x) = U_1(x) + iH_1(x)$ and $P_{nl}(x) = U_2(x) + iH_2(x)$ portrait the complex linear and nonlinear potentials, respectively. The point to be emphasized is that the PT-symmetry implies two constraints on the complex potentials, that are:

- The real parts of complex potentials are even functions, i.e; $U_l(x) = U_l(-x)$ for $l = 1, 2$.
- The imaginary parts of complex potentials are odd functions, i.e; $H_l(x) = -H_l(-x)$ for $l = 1, 2$. Ultimately the last term represents the nonlinearity [40–58].

2. Theoretical analysis

In this section, we are interested to obtain exact soliton solutions of Eq. (1) by using powerful integration tool, known as inverse engineering scheme [21,59–61].

In order to obtain the analytical soliton solutions of Eq. (1), we first choose the following ansatz

$$q(x, z) = B(x)e^{\left(i\left[\lambda z+\int f(x)\mathrm{d}x\right]\right)}. \tag{2}$$

In Eq. (2), $B(x)$ represents the real amplitude, $f(x)$ stand for inhomogeneous phase of the mode and λ denote the propagation constant.

Substituting Eq. (2) in Eq. (1) and separating real and imaginary parts, we get

$$\frac{1}{2}\mathrm{m}\,\mathrm{B}\frac{d^2B}{\mathrm{dx}^2} + \frac{1}{2}m(m-1)\left(\frac{\mathrm{d}\,\mathrm{B}}{\mathrm{dx}}\right)^2 - \left[\frac{1}{2}m^2f^2(x) + m\lambda - U_1(x)\right]B^2 + U_2(x)B^4 + F(B^2)B^2 = 0, \tag{3}$$

and

$$f(x) = -\frac{2}{B^{2m}}\int(B^{2m}[H_2(x)B^2 + H_1(x)])\mathrm{dx}. \tag{4}$$

The integration constant in Eq. (4) is zero. In order to obtain exact soliton solutions of Eq. (1), we will proceed by using the following steps [60,61].

- **Step 1.** Extract the phase gradient $f(x)$ from Eq. (4) for the known functions $H_1(x)$, $H_2(x)$ and $B(x)$. Mean while, the exact form of solution Eq. (2) is obtained.
  - Select $B(x) = B_0 sech(x)$ for bright soliton solutions, $B(x) = B_0 tanh(x)$ for dark soliton solutions and $B(x) = B_0 coth(x)$ for singular soliton solutions. Here, B_0 indicates the power of the real soliton amplitude $B(x)$.
  - The imaginary components $H_1(x)$ and $H_2(x)$ of PT-symmetric potentials should be selected as odd functions.
- **Step 2.** We extract the relation between $U_1(x)$ and $U_2(x)$ by putting $f(x)$ and $B(x)$ into Eq. (3).
  - We extract $U_1(x)$ if $U_2(x)$ is known and vice versa.
  - The real components of PT-symmetric mixed linear and nonlinear potentials ($U_l(x)$, $l = 1, 2$) should be even functions.

2.1. Kerr law

In this subsection, we are keen to extract soliton solutions of Eq. (1) in Kerr law media with PT-symmetric mixed linear and nonlinear optical complex lattices. For Kerr law, $F(B^2) = B^2$.

Then, Eq. (3) takes the form

$$\frac{1}{2}\mathrm{m}\,\mathrm{B}\frac{d^2B}{\mathrm{dx}^2} + \frac{1}{2}m(m-1)\left(\frac{\mathrm{d}\,\mathrm{B}}{\mathrm{dx}}\right)^2 - \left[\frac{1}{2}m^2f^2(x) + m\lambda - U_1(x)\right]B^2 + U_2(x)B^4 + B^4 = 0. \tag{5}$$

2.1.1. The imaginary components of complex linear and nonlinear potentials are the product of hyperbolic cosine and hyperbolic sine functions

Consider the following imaginary parts of the complex linear and nonlinear potentials: