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A modeling substorm dynamics of the magnetosphere using self-organized criticality approach

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ABSTRACT

Responses of Earth magnetic field during substorms exhibits a number of characteristics features such as the power-law spectra of fluctuations on different scales and signatures of low effective dimensions. Due the magnetosphere are constantly out-equilibrium their behavior is similar to real sandpiles during substorms, features of self-organized criticality (SOC) systems. Thus, in this work we presented a simple mathematical model to AE-index based on self-organizing sandpile mentioned by Uritsky and Pudovkin (1998), but we input the energy dissipation process inside the model. The statistical and multifractal tools to characterization of dynamical processes were used. The results also were compared with results from a classical geomagnetic event of the July, 2000, including the Bastille Day intense geomagnetic storm on 15 July.

1. Introduction

Due the geomagnetic system receive input energy from the Sun constantly, it can be represented by a complex dynamic similar to self-organizing criticality (SOC) such as occur to sand pile phenomena. Some authors developed a simple mathematical model based on SOC in order to study the low frequency fluctuations of the AE-index in the magnetosphere [1]. One important result from these authors was the two-dimensional sandpile model shows the SOC can explain fractal dynamics of AE-index. The SOC subject was presented by Per Bak in your seminal work [2] to describe the interaction between fractal processes and spatial fractal structures in complex dynamical systems. Several authors found SOC features in different physical systems such as financial markets, human brain and global biosphere ([3] and references therein).

In this framework, analysis and interpretation of the AE-index is performed in order to characterize the SOC features in terms of an intermittent energy injection, on the magnetosphere. As known from the turbulence theory the intermittency leads to deviation from usual Kolmogorov [4] velocity structure functions and its main signature are the singularity spectra exponents, $f(\alpha)$, which represent a power-law scaling-free dependence [5]. For solar wind turbulence, the so-called multifractal p-model describes how solar wind energy can be distributed among scales following a multiplicative rescaling structure [6,7]. Actually, the multifractal analysis, in contrast to the traditional power spectrum analysis, have shown that the Hölder exponents, for local singularity, are time dependent showing that the flux energy at a given scale is not homogeneously distributed in time as in the traditional homogeneous $1/f^{-1}$ turbulent spectrum. A usual hypothesis is that the intermittency behavior is associated with the multifractal turbulence model [8]. This fact suggests that the fluctuations can be described by means of a multifractal scaling law which is associated with intermittency, then admitting that nonlinear and coherent processes can coexist [9–11]. The multifractal approach was used in high-latitude geomagnetic fluctuations (77.5° N, 69.2° W) resulting deviations from the multiplicative cascade p-model [12]. Our previous study [13], we had analyzed the variability pattern during the Bastille Day intense geomagnetic storm on 15 July, using the multifractal

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approach. The results had shown the occurrence of low-latitude multifractal processes driving the intermittent coupling between the magnetosphere and solar wind. Thus, the present study deals with the multifractal and statistical analyses of simulated geomagnetic H-components time series based on simple SOC model which is fully intermittent.

It is important to mention that Bastille Day refers to a severe geomagnetic storm event occurred on 15 July 2000, where this date was an important social and historical event take place in France, the Bastille Day. In this event, a halo Coronal Mass Ejection (CME), in which the solar wind velocity was above 1000 km/s and the Bz reached −59 nT, hits the magnetosphere where the minimum Dst was about −300 nT. Furthermore, this CME caused an suddenly increase in the auroral electrojet (AE) where this index reached above 3000 nT, causing an expansion of the aurora oval toward low latitudes ([14] and references therein).

2. Methodology

We used a mathematical model described by Uritsky and Pudovkin [1] but we done some important improvement. We also considered a two-dimensional self-organized model as a tool for triggered reconnections in the magnetosphere current sheet. Thus, we considered a rectangular grid 100 × 100 of elements where which one have a free magnetic energy, Z. In some time each element is characterized by energy $Z_t\,(x, y)$ and when $Z_t\,(x, y) < Z_c, Z_c$ is a critical threshold value chooses, the cell remains in equilibrium, but if this threshold is exceeded this excess of energy is distributed to neighboring cells randomly. Mathematically, we can express this modeling as following:

$$Z_{t+1}\,(x, y) = Z_t\,(x, y)\,; Z \le Z_c \tag{1}$$

$$Z_{t+1}\,(x, y) = Z_t\,(x, y) - 8 - D; \tag{2}$$

$$Z_{t+1}\,(x \pm 1, y \pm 1) = Z_t\,(x \pm 1, y \pm 1) + 1; Z > Z_c. \tag{3}$$

The equations before are almost the equations given by Uritsky and Pudovkin [1], but in our mathematical approach the cell with excess distribute it for eight neighbors around, not only four as used by Uritsky and Pudovkin [1]. Furthermore, we inserted a dissipation term, D, in Eq. (2), in order the system dissipate the energy as happens in the real scenario from the magnetosphere system.

As mentioned before, the SOC subject was developed by Bak et al. [2] in order to study fractals systems which exhibit some kind of chain reaction, i.e., a lack of stability of some cell can lead the system to a strong instability called avalanche. Thus, to study our mathematical model we used the multifractal approach.

We have considered, in order to obtain the singularity spectrum $f(\alpha)$ from the $F(t)$ time series from our model, the Wavelet Transform Modulus Maxima (WTMM) [8,15]. The basic idea behind the WTMM method is to describe a partition function over only the modulus maxima of the wavelet transform of a signal $H(t)$.

The wavelet transform of $H(t)$ is written as (Eq. (1), page 64 from Enescu et al. [16])

$$W_\psi[H](s, b) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} H(t)\, \psi \left[\frac{t-b}{s}\right] dt, \quad s > 0 \tag{4}$$

where s, b are real, $s > 0$ and ψ^* is the complex conjugate of a continuous wavelet function ψ. This transformation gives the coefficient of the wavelet decomposition of the signal $H(t)$ at time $t = b$ for scale s (e.g., [16]). For analysis where the variability pattern contain nonstationary power at many different scales, such as $H(t)$, a wavelet analysis based on a plane wave modulated by a Gaussian is required. Thus, it is then considered the Morlet wavelet, here taken in its form to satisfy the so-called admissibility condition [17,18]:

$$\psi(t) = \pi^{-1/4} e^{i\omega t} e^{-t^2/2}, \tag{5}$$

where $\omega \ge 5$.

The scaling and translation of this *mother* wavelet function over the signal $H(t)$ are performed by the parameters s and b. While the scale parameter s stretches (or compresses) the mother wavelet to the required resolution, the translation parameter b shifts the basis functions to the desired time location.

It can be shown that the wavelet transform can reveal the local characteristics of $H(t)$ at a point t_0. More precisely, we have the following power-law relation (Eq. (4), page 64 from Enescu et al. [16]:

$$W_\psi[H]\,(s, t_0) \approx |s|^{\alpha(t_0)}, \tag{6}$$

where $\alpha(t_0)$ is the Hölder exponent (or *singularity strength*). Thus, the exponent $\alpha(t_0)$, for fixed location t_0, can be obtained from a log–log plot of the wavelet transform amplitude versus the scale s. However, this power-law characterization is difficult when the process is governed by a hierarchical distribution of singularities compromising the exact determination of α on a finite range of scales. In such case any transformation of the signal $H(t)$ may obey some renormalization operation involving multiplicative cascades and it has been demonstrated that the local maxima of $|W_\psi\,(s, b)|$ at a given scale s, are likely to contain all the hierarchical distribution of singularities in the signal. At a given scale s each one of the WTMM bifurcates into new two maxima giving rise to a rich multiplicative cascade in the limit $s \to 0$. Thus, it is possible to identify a