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A dynamic equilibrium model of the oligopolistic closed-loop supply chain network under uncertain and time-dependent demands

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ABSTRACT

In this paper, we develop a dynamic equilibrium model of oligopolistic closed-loop supply chain network to account for the seasonality of demand. In this model, demands and returns are uncertain and time-dependent. The dynamic Cournot-Nash equilibrium of the oligopolistic network is constructed by evolutionary variational inequality and projected dynamical systems. The existence and monotonicity results are established and a computational method is developed for finding the dynamic Cournot-Nash equilibrium. Numerical examples are solved to illustrate the performance of our model. The results show that the firms' optimal production and transaction quantities are strongly affected by the time-dependent demands.

1. Introduction

Nowadays, more firms are aware of the importance of integrating a supply chain as a whole, consisting of all the marketing activities of all competitors. The integration of the entire supply chain generates oligopolistic competition among firms. Thus, a lot of oligopolistic competition supply chain network (SCN) model have been developed. Masoumi et al. (2012), Nagurney and Yu (2012) and Yu and Nagurney (2013) developed, respectively, oligopolistic competition network models involving the optimal management of pharmacies, fashion products and fresh food, all of which are perishable products. Nagurney and Li (2014) considered an oligopolistic competition network model involving product differentiation and quality competition. Li and Nagurney (2015), Nagurney et al. (2016) and Masoumi et al. (2017) developed, respectively, oligopolistic competition equilibrium models for competing suppliers, post-disaster humanitarian relief and blood banking systems. Nagurney et al. (2017) considered capacity competition in an oligopolistic competition network. Nagurney and Shukla (2017) developed three distinct equilibrium models for a cybersecurity investment problem. Research work on the closed-loop supply chain (CLSC) oligopolistic competition equilibrium model began only recently. Zhou et al. (2014) studied an oligopolistic equilibrium model for a CLSC network involving uncertain demand and return. All the above models analyzed the static oligopolistic competition equilibrium problem.

However, many phenomena of our economic and physical world are complicated dynamical systems, because the demand of product changes with time. For time-dependent deterministic network, Barbagallo and Cojocaru (2009) considered a dynamic oligopolistic equilibrium problem in which all the given data are time-dependent. They established the continuity and regularity results for solving the dynamic equilibrium. Barbagallo and Mauro (2012) considered an oligopolistic market equilibrium problem involving both production and demand excesses. By using an appropriate evolutionary (time-dependent) variational inequality (EVI), some

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existence and regularity results for equilibrium solutions were established. Barbagallo and Mauro (2012) generalized a previous model of the dynamic oligopolistic market equilibrium problem allowing the presence of production excesses. Feng et al. (2014) focused on a CLSC network competition problem with time-parameter. Nagurney et al. (2014) focused on a SCN oligopolistic competition problem in which firms produce timely deliveries of products. Their model captures the time associated with the various SCN activities of manufacturing, transportation, storage, and ultimate distribution to the demand markets. Warburton et al. (2014) gave the exact solutions for the supply chain equations with time-dependent demands. Altendorfer (2017) considered time dependent demand and capacity flexibility in a two-stage supply chain. Guner et al. (2017) considered stochastic time-dependent networks for milk-run tours with time windows. Sun et al. (2018) considered a time-dependent capacitated profitable tour problem. The above researches involving the dynamic equilibrium problems focused only on the deterministic network.

In contrast to the existing oligopolistic competition equilibrium models, we construct a dynamic equilibrium model of the oligopolistic CLSC network in this paper, which is modified from the model of Zhou et al. (2014) by considering the time-varying demand and return. In view of the fact that the demand for a lot of products, such as electronics, toy, fan, heater, etc. is highly seasonal (Ross et al., 2008), we therefore focus our study on the demand of a highly seasonal product in this paper. Thus, we assume that the demand and return are uncertain and time-dependent, whose expected values are simple cosine functions of time. In this way, we can model the demand in such a way that there is a greater demand in summer and a lesser demand in winter, thus the seasonality of the demand is established. As a consequence, the production costs, the shipment costs, the storage costs and the shortage costs all depend on time. In other words, we extend the results of the oligopolistic competition model of Zhou et al. (2014) from 'static equilibrium problem' to 'dynamic equilibrium problem'.

Moreover, we formulate the oligopolistic competition problem of our model as a dynamic game, which is very similar to an infinitely repeated simultaneous move game problem. In contrast to the conventional differential game problem, the evolution of the state in our model is not governed by differential equations. We solve the oligopolistic competition problem of each manufacturer by using a continuous nonlinear program approach instead of an optimal control approach. We establish the EVI formulation of our problem and then solve it by the projected dynamical systems (PDS) method (introduced by Dupus and Nagurney (1993)). This nonlinear programming approach is very similar to those used in solving real-life dynamic Nash equilibrium problems (Daniele et al., 1999; Daniele, 2004; Raciti and Scrimali, 2004; Cojocaru et al., 2005; Daniele, 2006; Li et al., 2007; Daniele, 2010). On the other hand, differential game problems are special dynamic game problems whose system dynamics are described by differential equations. Optimal control methods have been used by a lot of researchers for solving differential game problems. More recently, Dockner et al. (2000) gave an introduction to the theory of differential games and their applications. Jorgensen and Zaccour (2007) discussed the applications of differential games in economic and management science. They also established the analytical results of differential games by using Hamilton-Jacobi-Bellman equation or maximum principle. Chen and Sheu (2009) established a differential game model for the sales competition and recycling dynamics problem and solved these game problems also by using the Hamilto-Jacobi-Bellman equation. Friesz (2010) discussed the traditional methods in optimal control theory, such as the Pontryagin maximum principle and Hamilton-Jacobi-Bellman, for solving differential game problems. Gwinner (2013) used the differential variational inequalities (introduced by Pang and Stewart (2008)) to solve a dynamic equilibrium problem. Xin and Sun (2018) considered a differential game model for a production planning and water savings problem. All the differential game problems in the above research stream are solved by the traditional methods in optimal control theory, whereas, as mentioned earlier, the repeated simultaneous move game problem of our model is solved by traditional method in nonlinear programming. Thus, the oligopolistic competition game of our model and the conventional differential game use different approaches for tackling the problems.

Lastly, we use a gradient method given in Marcotte (1991) to find the critical points of the projected dynamical system (PDS), which provides us with the equilibrium decision making.

The organization of our paper is as follows. Section 2 develops the dynamic equilibrium model of an oligopolistic CLSC network with multi-products, uncertain demands and returns and constructs the dynamic equilibrium conditions. Section 3 establishes the EVI for the dynamic equilibrium of our model. Section 4 establishes the existence and monotonically results of the EVI and provides the PDS associated with the EVI in Section 3. In Section 5, we use a gradient method to find the critical points of the PDS. In Section 6, we provide a differential game formulation for our dynamic Cournot-Nash equilibrium model. In Section 7, we solve numerical examples to illustrate the performance of our model. Concluding and suggestions for further studies are given in Section 8.

2. A dynamic equilibrium model of an oligopolistic CLSC network

In this section, we construct a dynamic equilibrium model of a CLSC network involving oligopolistic competition among firms. This model is modified from the static model of Zhou et al. (2014) by allowing the demand and the return to follow a probability function which is time-varying. As a consequence, the production costs, the shipment costs, the storage costs and the shortage costs all depend on time. For the sake of convenience of the readers, we shall describe our model as follows:

The CLSC network consists of I oligopolistic firms competing non-cooperatively in a dynamic environment. The planning horizon of our problem is $[0, T]$. The topology of the network of this model, which is the same at any instant $t \in [0, T]$, is as shown in Fig. 1.

Each firm i $(i = 1, ..., I)$ produces J products at any time t. The demand for products is time varying, i.e., we may have a greater demand in summer and a lesser demand in winter. In order to satisfy the demand, the firms either manufacture new products from virgin material or from used material through recycling. Both the demands and returns are random variables. The problem is to determine the production quantities of each new product from virgin material as well as the amount of the product flows in the forward supply chain at each instant t so that all the oligopolistic firms' expected profit is maximized in the planning horizon $[0, T]$.

In the CLSC network, each firm is represented as a network of its economic activities. In the forward supply chain, firm i