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Pricing of general European options on discrete dividend-paying assets with jump-diffusion dynamics

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ABSTRACT

Stocks regularly pay dividends at discrete intervals of time while statistical evidence indicates the existence of small "jumps" in the stock price dynamics. In this paper, we find closed-form solutions for the valuation of European options when the underlying asset is modeled by a jump-diffusion process and pays discrete or continuous dividends. The formula is very general and can be used with any specification on the distribution of the jump. Moreover, the formula is written in terms of the Black–Scholes formula with no jumps or dividends and thus indicates the effect of the jumps and the effect of the inclusion of discrete (or continuous) dividends on the price of the option.

1. Introduction

Assets such as equities pay dividends, which are payments to shareholders out of a company's profits. In reality, dividends are paid at discrete times; usually after the disclosure of the amount of the dividend. Quite often these payments are simply ignored in option valuations, although the price of an option on a dividend-paying underlying asset depends critically on the dividend payments. To model dividend payments, the amount and timing of the payments need to be considered. Individual companies typically make two to four payments in a year, usually after the company finalizes its income statement, in which case dividend payments essentially need to be treated discretely. It is only when the dividend payments are very frequent, such as on an index like the S&P 500, that dividend payments can be approximated by a continuous payment. Here we assume only deterministic dividends whose amount and timing are known at the start of the option's life. As for the structure of dividend payments, we will incorporate both discrete and continuous payments.

Modified asset price models that incorporate dividend payments have been considered in the literature. Hull [1, p. 298] subtracted the net present value of all dividends in the option's lifetime from the current asset price, and the option price can then be determined once this adjustment is made. Musiela and Rutkowski [2, pp. 53–54] modified the Cox–Ross–Rubinstein model [3] and added to the strike price the future value at maturity of all dividends paid during the lifetime

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of the option. In an attempt to reconcile the results in [1] and [2], Bos and Vandermark [4] provided a method that splits multiple dividend payments into two categories: "near" (i.e., dividend payments about to occur) and "far" (i.e., dividend payments close to expiry). The idea was to subtract the "near" dividends from the underlying's value and add "far" dividends to the strike price.

Wilmott et al. [5] considered a model that assumes geometric Brownian motion with jumps at the dividend payment dates and studied it numerically. They also provided a method to unify continuous and discrete dividend payments, and this is the approach that will be pursued in this article. Björk [6, Chapter 15] also gives a description of the European option pricing problem with continuous and discrete dividend-paying assets. In particular, a closed formula for the option price is given for proportional dividends.

Veiga and Wystup [7] used a Taylor series expansion to derive a closed pricing formula for a European call on an asset that pays discrete dividends under the Black–Scholes framework, which yields formulas for its partial derivatives. The drawback of their method is that the Taylor series expansion is not always convergent. By applying a semidiscretization technique on the asset, Ballester et al. [8] constructed numerical solutions of the Black–Scholes equation incorporating a discrete dividend payment.

Most of the above results take as a starting point the Black–Scholes model for the asset price, which has continuous sample paths, and then include the effect of dividend payments. While the geometric Brownian motion assumed by the Black–Scholes model is convenient, it cannot capture many of the features of stock price returns, such as the skew (or smile) features of the implied volatility surface.

In the absence of dividend payments, Merton [9] considered a jump-diffusion process that allows for the probability of the asset price to change at large magnitudes irrespective of the time interval between successive observations. The jumps in the asset price can be accommodated by including an additional source of uncertainty into the asset price dynamics. Subsequent empirical studies have asserted that the asset price is best described by a process with a discontinuous sample path [10–13]. Compared to diffusion models, jump-diffusion models produce rich structures of the distribution of asset returns and implied volatility surfaces (see Cont and Tankov [14]). The main objective of this article is to price European options *when the underlying asset is modeled by a jump-diffusion process AND pays discrete dividends*, which to the authors' knowledge has not been considered previously. Moreover, instead of a probabilistic approach, a partial integro-differential equation (PIDE), which can be difficult to solve numerically, is solved analytically using Mellin transform techniques.

For definiteness, take a European option with expiry T on a dividend-paying asset. Denote the asset and option prices at time t by S_t and V_t, respectively. Assume that $V_t = v(S_t, t)$ for some deterministic function $v = v(x, t)$. Suppose that the asset pays n dividends, say at times $t_1, \dots, t_n$, where $0 < t_1 < \cdots < t_n < T$, during the lifetime of the option. At each t_j, where $j = 1, \dots, n$, let $0 \le q_j \le 1$ be a known constant dividend yield. Therefore, in the absence of other factors, the asset price falls by exactly the amount of the dividend payment, i.e., $S_{t_j+} = S_{t_j-} - q_j S_{t_j-} = (1 - q_j) S_{t_j-}$. It can be shown [5] that the option price must satisfy the jump condition $V_{t_j-} = V_{t_j+}$ for any realization of the asset price. In terms of the function v, this is equivalent to requiring that

$$v(x, t_j-) = v((1 - q_j)x, t_j+), \quad x \in \mathbb{R}_+ = (0, \infty). \tag{1.1}$$

To unify continuous and discrete dividend payments [5], the Black–Scholes asset price stochastic differential equation (SDE) is modified to

$$\mathrm{d}S_t = [r - D(t)]S_t\,\mathrm{d}t + \sigma S_t\,\mathrm{d}W_t, \tag{1.2}$$

where $S = \{S_t : t \in [0, T]\}$ is the asset price process, $W = \{W_t : t \in [0, T]\}$ is a Wiener process with respect to the risk-neutral measure, r is the constant riskless interest rate, σ is the constant volatility, and $D(t)$ is the (deterministic) dividend yield at time t. For continuous payments, D is a continuous function of t with $D(t) \in [0, 1]$, while for discrete payments, $D(t) = \sum_{j=1}^{n} D_j \delta(t - t_j)$, where δ is the Dirac delta function and $\mathrm{e}^{-D_j} = 1 - q_j$ for all $j = 1, \dots, n$.[1] Let a non-dividend-paying asset price process $\bar{S} = \{\bar{S}_t : t \in [0, T]\}$ be described by the usual Black–Scholes model

$$\mathrm{d}\bar{S}_t = r\bar{S}_t\,\mathrm{d}t + \sigma\bar{S}_t\,\mathrm{d}W_t, \tag{1.3}$$

and let $\bar{V}_t = \bar{v}(\bar{S}_t, t)$ be a European option with expiry T on the asset $\bar{S}$, where $\bar{v} = \bar{v}(\bar{x}, t)$ is some deterministic function. Suppose that $\bar{v}$ can be determined; hence $\bar{V}_t = \bar{v}(\bar{S}_t, t)$ is known. It makes sense to try to use the option price $\bar{V}_t$ on a non-dividend-paying asset described by (1.3), which is the solution to an easier problem, to determine the option price V_t on an asset that pays dividends described by (1.2).

To account for the possibility of instantaneous jumps in the asset price, Merton [9] proposed the following modification of (1.2) by assuming that the discontinuous jumps arrive as a Poisson process (here we incorporate dividend payments):

$$\mathrm{d}S_t = [r - D(t) - \lambda E(Y - 1)]S_t\,\mathrm{d}t + \sigma S_t\,\mathrm{d}W_t + (Y - 1)S_t\,\mathrm{d}N_t, \tag{1.4}$$

where Y is a nonnegative continuous random variable with $Y - 1$ denoting the impulse change in the asset price from S_t to YS_t as a consequence of the jump, E is the expectation operator, and $N = \{N_t : t \in [0, T]\}$ is a Poisson process with constant intensity λ and such that $\mathrm{d}N_t = 1$ (respectively, $\mathrm{d}N_t = 0$) with probability $\lambda\,\mathrm{d}t$ (respectively, $1 - \lambda\,\mathrm{d}t$). Furthermore, it is

[1] Section 4 gives a derivation of this result, which is a consequence of the jump condition (1.1), under a more general setting with jumps.