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A reduced order accelerator for time-dependent segregated neutronic solvers

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ABSTRACT

The deterministic solution of the neutron transport problem entails the coupled solution of several partial differential equations, one for each energy group, direction and/or spherical harmonic. Several techniques have been devised for accelerating the solution of this set of equations, both for time dependent and eigenvalue calculations. This paper describes an acceleration technique based on reduced order models and applicable to the segregated solution of time dependent solutions. In this work the technique is applied to the simple case of multi-group diffusion and tested on two cases of practical interest. It shows performances that are comparable to some commonly employed acceleration techniques. Some potential advantages have been observed for transients with significant flux deformations. In addition, possibly interesting features of the proposed technique are: a relatively easy implementation in general PDE solvers and numerical libraries; its potential applicability to any kind of problem requiring the iterative solution of a system of equations; a flexible implementation with a wide margin for possible modifications.

1. Introduction

The space, energy and angular dependency of neutron population in nuclear reactors is governed by a Boltzmann transport equation. Such equation cannot be solved directly based on deterministic methods and approximations are introduced for treating the distribution of neutrons. The most typical angular approximations are an expansion in spherical harmonics, a discretization of the solid angle (SN discrete ordinate methods), or the assumption of an isotropic behavior of neutrons (diffusion approximation). In addition, the energy dependency of neutrons is normally treated by grouping them into different energy groups (multi-group approach). These approximations can dramatically increase the number of equations to be solved, for which a coupled solution for time dependent problems is often obtained through iteration. Due to the stiffness of the system, several tens or hundreds of iterations are normally required at each time step.

A possible trivial solution to the convergence problem would be to couple in the same matrix the equations for the different angular and energy groups. An example of this for the case of multi-group diffusion can be found in the work by Clifford and Jasak (2009). However, the resulting matrix is often badly conditioned in transport problems, and it is large and sparse even in the simple case of diffusion (Bru et al., 2002). This technique is then rarely employed.

Several other techniques have been proposed in the past in the attempt to reduce the number of iterations required to solve this system of equations. A simple method consists in applying a series acceleration algorithm to the set of fluxes obtained during the iterative process. Typical examples are the application of the Aitken's delta-squared method (Press et al., 2007) and its more generalized version, the Wynn Epsilon method (Wynn, 1962). These methods are agnostic to the physical nature of the problem and are based on a prediction of the flux at the next iteration based on the previous iterations (Mahadevan et al., 2012). Other, more complex techniques exist that attempt a series acceleration based on the spectral characteristic of the iteration matrix. This is for instance the case of the Modified Fission Source Iteration (Zimin and Schukin, 1992) and the Chebyshev-based acceleration techniques like the Chebyshev Semi-Iterative method (Varga, 1962) and the Chebyshev Semi-Analytical method (Zimin and Ninokata, 1996). Variational techniques have also been employed e.g. by Ginestar et al. (1998).

A different approach for accelerating the convergence of the neutron transport problem consists in the introduction of a predictor step. The objective is to start iterations from a solution that is closer to the converged one. A trivial physics-based prediction step consists in an integral neutron balance to predict the average change in neutron population. Another possibility is an interpolation based on previous time steps. Finally, an effective technique

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Nomenclature

Latin symbols

$\overline{\overline{A}}_i$	matrix resulting from the finite volume discretization of the diffusion equation for the i^{th} energy group [m^{-1}]
$\overline{\overline{A}}_{R,i}$	matrix resulting from the Galerkin projection of matrix $\overline{\overline{A}}_i$ [m^{-1}]
$\bar{b}_i$	source term resulting from the finite volume discretization of the diffusion equation for the i^{th} energy group (Eq. (7)) [$m^{-3}\cdot s^{-1}$]
$\bar{b}_{R,i}$	source term resulting from the Galerkin projection of the source term $\bar{b}_i$ (Eq. (8)) [$m^{-3}\cdot s^{-1}$]
$\bar{c}_i, c_{i,k}$	vector of coefficients, or k^{th} coefficient, for flux reconstruction for the i^{th} energy group (Eqs. (6) and (9)) [−]
D_i	neutron diffusion coefficient for the i^{th} energy group[m]
$\overline{\overline{I}}$	identity matrix [−]
k_{eff}	effective multiplication factor [−]
S_d	delayed neutron source [$m^{-3}\cdot s^{-1}$]
$S_{n,i}$	fission neutron source from neutron energy groups others than the i^{th} [$m^{-3}\cdot s^{-1}$]
$S_{s,i}$	scattering neutron source from neutron energy groups others than the i^{th} [$m^{-3}\cdot s^{-1}$]
t	time [s]
v_i	average neutron velocity for the i^{th} energy group [$m\cdot s^{-1}$]

Greek symbols

β_t	total delayed neutron fraction [−]
υ	average number of neutrons per fission [−]
$\Sigma_{r,i}$	removal (disappearance) cross section for the i^{th} energy group [m^{-1}]
$\Sigma_{f,i}$	fission cross section for the i^{th} energy group [m^{-1}]
φ_i	neutron flux for the i^{th} energy group [$m^{-2}\cdot s^{-1}$]
$\chi_{d,i}$	delayed neutron yield for the i^{th} energy group [−]
$\chi_{p,i}$	prompt neutron yield for the i^{th} energy group [−]
Ψ_i	basis function for group reconstruction for the i^{th} energy group [−]

The cap, as in $\bar{S}_{n,i}$ or $\overline{\overline{D}}_i$, indicates the discretized form of a variable or an operator (vector or matrix, respectively). The superscripts "l" and "n" indicates l^{th} intra-step iteration and n^{th} time step, respectively. The subscript "i" indicates the i^{th} energy group.

consists in the implicit solution, for each cell of the domain, of the full system of equations with explicit spatial terms (Fiorina et al., 2016). It should be noted that predictor steps and series acceleration methods can easily and effectively be coupled in the same solver.

A special case is then represented by methodologies that employ simplified problems to provide a better estimate of the solution at each iteration step. This strategy resembles that of the above-mentioned predictor step, with the main difference that in this case the prediction-correction procedure is repeated at each iteration. The most typical example of this kind of methods is the diffusion synthetic acceleration (Larsen, 1984). This acceleration method is often employed for discrete-ordinate problems and consists in solving, for each transport sweep, a diffusion-like problem that is used to adjust the current iterate in order to improve the solution of the next iterate.

In this paper, an acceleration method is proposed that attempts a faster solution neither via a predictor step, nor by reducing the number of iterations. The idea is to accelerate the solution process for each iteration by solving a (mathematically) reduced order problem obtained via Reduced Order Modelling (ROM) techniques. This accelerator can be combined with the use of a predictor step. In principle, it may also be combined with previously mentioned techniques for series acceleration, but this would require a careful investigation for each specific case.

Although in principle the proposed methodology is applicable to all cases that require an iterative solution of a system of equations, this work will focus on the application of this methodology to the case of diffusion. The paper is structured as follows. In Section 2 a physical and mathematical description of the problem is provided. In Section 3, the proposed acceleration technique is described. Section 4 evaluates the performances of the proposed technique based on two examples of practical interest. The conclusions of the paper are drawn in Section 5.

2. Description of the problem

The angular and energy approximations of the neutron transport equation lead to a system of linear partial differential equations that can be discretized using different techniques (Hebert, 2009) to obtain a set of matrix equations for each energy group i and for each direction or spherical harmonic. As a test case, in this paper we focus on the multigroup diffusion equations (Stacey, 2007):

$$\frac{1}{v_i}\frac{\partial \varphi_i}{\partial t} = \nabla\cdot D_i\nabla\varphi_i + \frac{\upsilon\Sigma_{f,i}(1-\beta_t)\chi_{p,i}}{k_{eff}}\varphi_i - \Sigma_{r,i}\varphi_i + \frac{S_{n,i}(1-\beta_t)\chi_{p,i}}{k_{eff}} + S_d\chi_{d,i} + S_{s,i} \quad (1)$$

This equation can be discretized, for each time step n and iteration l, as:

$$\frac{(\overline{\varphi}_i)^{n+1,l}}{v_i\Delta t}\overline{\overline{I}} - \overline{\overline{D}}_i(\overline{\varphi}_i)^{n+1,l} - \overline{\overline{I}}\frac{\upsilon\Sigma_{f,i}(1-\beta_t)\chi_{p,i}}{k_{eff}}(\overline{\varphi}_i)^{n+1,l} + \overline{\overline{I}}\Sigma_{r,i}(\overline{\varphi}_i)^{n+1,l} = \frac{(\overline{\varphi}_i)^n}{v_i\Delta t} + \left(\frac{\bar{S}_{n,i}(1-\beta_t)\chi_{p,i}}{k_{eff}} + \bar{S}_{s,i} + \bar{S}_d\chi_{d,i}\right)^{n+1,l-1} = \frac{(\overline{\varphi}_i)^n}{v_i\Delta t} + (\bar{S}_i)^{n+1,l-1} \quad (2)$$

More precisely, in this work the iteration follows a Gauss-Seidel scheme, where each source term is recalculated as soon as the corresponding flux is available from the current iteration.

Eq. (3) generally features an extremely slow convergence rate within each time step. From a mathematical perspective this is caused by a spectral radius of the coupled iteration matrix being very close to unity (Zimin and Ninokata, 1996). From a more heuristic perspective, one can notice that if an instantaneous reactivity insertion is simulated, this will translate into the term $\bar{S}_i$ being increased. The overall source term $\frac{(\overline{\varphi}_i)^n}{v_i\Delta t} + (\bar{S}_i)^{n+1,l-1}$ will then be increased by a lower amount, since $\frac{(\overline{\varphi}_i)^n}{v_i\Delta t}$ is constant (calculated from fluxes at the previous time step). Since every term in the left-hand side of Eq. (3) is proportional to the flux, the latter will be increased by the same relative amount as the source term. The term $(\bar{S}_i)^{n+1,l-1}$ will then be updated and the iteration will continue until the relative change in the fluxes is sufficiently small. By doing some tests one can notice that in this process the convergence rate depends on the ratio between the constant part of the source term $\frac{(\overline{\varphi}_i)^n}{v_i\Delta t}$ and the part which is instead updated at every iteration ($(\bar{S}_i)^{n+1,l-1}$). Such a ratio is typically very small in practical