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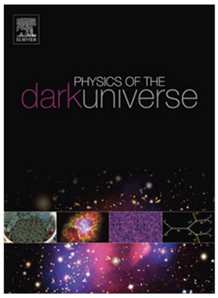

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Derivation of a generalized Schrödinger equation for dark matter halos from the theory of scale relativity

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Using Nottale's theory of scale relativity, we derive a generalized Schrödinger equation applying to dark matter halos. This equation involves a logarithmic nonlinearity associated with an effective temperature and a source of dissipation. Fundamentally, this wave equation arises from the nondifferentiability of the trajectories of the dark matter particles whose origin may be due to ordinary quantum mechanics, classical ergodic (or almost ergodic) chaos, or to the fractal nature of spacetime at the cosmic scale. The generalized Schrödinger equation involves a coefficient $\mathcal{D}$, possibly different from $\hbar/2m$ (where $\hbar$ is the Planck constant and m the mass of the particles), whose value for dark matter halos is $\mathcal{D} = 1.02\times 10^{23}\,{\rm m^2/s}$. This model is similar to the Bose-Einstein condensate dark matter model except that it does not require the dark matter particle to be ultralight. It can accomodate any type of particles provided that they have nondifferentiable trajectories. We suggest that the cold dark matter crisis may be solved by the fractal (nondifferentiable) structure of spacetime at the cosmic scale, or by the chaotic motion of the particles on a very long timescale, instead of ordinary quantum mechanics. The equilibrium states of the generalized Schrödinger equation correspond to configurations with a core-halo structure. The quantumlike potential generates a solitonic core that solves the cusp problem of the classical cold dark matter model. The logarithmic nonlinearity accounts for the presence of an isothermal halo that leads to flat rotation curves (it also accounts for the isothermal core of large dark matter halos). The damping term ensures that the system relaxes towards an equilibrium state. This property is guaranteed by an H-theorem satisfied by a Boltzmann-like free energy functional. In our approach, the temperature and the friction arise from a single formalism. They correspond to the real and imaginary parts of the complex friction coefficient present in the scale covariant equation of dynamics that is at the basis of Nottale's theory of scale relativity. They may be the manifestation of a cosmic aether or the consequence of a process of violent relaxation and gravitational cooling on a coarse-grained scale.

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I. INTRODUCTION

The nature of dark matter (DM) is still unknown and remains one of the most important open problems of modern cosmology. The existence of DM has been predicted by Zwicky in 1933 to account for the missing mass of the galaxies in the Coma cluster inferred from the virial theorem [1]. The robust indication of DM came later from the measurement of the rotation curves of spiral galaxies [2–5] that revealed that they were flat instead of declining with the distance like in the case of planetary systems that are dominated by a central mass (Kepler's law). The existence of DM has been confirmed by independent observations of gravitational lensing [6], hot gas in clusters [7], and the anisotropies of the cosmic microwave background (CMB) [8]. Very recently, astronomers reported that Dragonfly 44, an ultra diffuse galaxy with the mass of the Milky Way but with no discernable stars may be made almost entirely of DM [9].

Observation of the large-scale structure (LSS) of the Universe and the CMB are consistent with the cold dark matter (CDM) model in which DM is modeled as a pressureless gas described by the Euler-Poisson equations or as a collisionless system described by the Vlasov-Poisson equations. The most studied candidate particles for DM are WIMPs (weakly interacting massive particles) with a mass in the GeV-TeV range. These particles are the lightest supersymmetric partners predicted by models of supersymmetry (SUSY) [10]. The CDM model works remarkably well at large (cosmological) scales and is consistent with ever improving measurements of the CMB from WMAP and Planck missions [11, 12]. It is able to account for the formation of structures, with the small objects forming first and merging over time to form larger objects (hierarchical clustering). This leads to a "cosmic web" made of virialized halos connected by filaments delimiting empty voids, in very good agreement with observations. However, the CDM model experiences serious difficulties at small (galactic) scales. In particular, being pressureless, numerical simulations of CDM lead to cuspy density profiles [13], with a density diverging as r^{-1} for $r\to 0$, while observations favor cored density profiles with a finite density at the center [14]. This is the "cusp-core" problem [15]. Other related problems are known as the "missing satellites" problem [16–19] and the "too big to fail" [20] problem. The expression "small-scale crisis of CDM" has been coined.[1]

[1] Some researchers remain unconvinced that there is a real problem at the center of the galaxies. For example, the cusp-core problem could be an effect of asphericity and misalignment of the halos. We refer to [21] for a detailed discussion of this issue and additional references.