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Riemann surface of dispersion diagram of a multilayer acoustical waveguide

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HIGHLIGHTS

- Precursors in a three-layer acoustic waveguide are studied using an analytical continuation of the dispersion diagram.
- The structure of the Riemann surface of the dispersion diagram is revealed.
- A numerical procedure for finding the branch point of the dispersion diagram is proposed.

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ABSTRACT

A Riemann surface of a dispersion diagram of a multilayer planar waveguide is studied. Structure of this surface is linked with the transient wave phenomena in the waveguide. A method is proposed to study the dispersion diagram. The method is constituted in introducing of an artificial parameter controlling the links between the layers of the waveguide. The strength of the links varies from the full isolation to the realistic physical contact. The dependence of the solution on the artificial parameter is analytical. Using this approach it is proven that the branch points of the Riemann surface can be listed as the crossing points of the branches of non-interacting layers. A numerical procedure of finding the branch points positions is described. An explanation of the types of waves corresponding to different parts of the Riemann surface of the dispersion diagram is given.

1. Introduction

Multilayer acoustic waveguides are of great importance for geological [1], oceanic [2], and medical physics [3,4]. The simplest layered waveguide is a single layer on a half-plane substrate. Such a problem has been introduced in [2]. In later works [5–8] it has been shown that the wave field in such waveguides consists of so-called proper modes, leaky waves and head waves. In [9,10] the case of an n-layer waveguide with half-infinite first and last layers has been studied. The formal modal series solution has been obtained, and an analogy with elastic plate waveguide has been indicated. Using a semi-intuitive arguments it has been shown that modes close to leaky waves can exist in such waveguides. Also, the same result has been proven numerically for the three-layer waveguide with compact cross-section. In recent experimental works [11,3,4] it has been shown that in some layered structures one can observe exponentially decaying precursor signals. In [12], which is deeply based on [13,14], these precursors have been studied in the case of two-layer waveguide. In recent theoretical papers [15,16] transient phenomena in layered waveguides has been studied. Some expressions related to the inflection points of the dispersion diagram (closely related to the avoiding-crossings studied in [12]) are obtained.

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Note that there is a considerable difference between waveguides comprising a semi-infinite substrate and waveguides of a finite cross-section. Physically, the difference is explained by the existence of waves that can evacuate energy far from the surface of the waveguide in the first case. In applications, the waveguides with a substrate describe various geophysical situations (the substrate is the sea bottom, for example). The waveguides with a finite cross-section emerge in defectoscopy and medical physics. The current paper considers waveguides with a finite cross-section.

The current paper is a continuation of [12]. The key idea of [12] is that transient phenomena in layered waveguides can be described with the help of an analytic continuation of the dispersion diagram into the domain of complex temporal and spatial frequencies. Namely, a standard representation of the field is a series–integral expression. The series is taken over the modes of the waveguide, while the integral is taken over the frequency axis ω. One can deform the integration contour in the complex plane. While the contour is deformed, it crosses some branch points of the dispersion diagram. After such crossings, the structure of the integrand on the contour becomes simpler and the properties of the field can be revealed.

Thus, the structure of the Riemann surface of the dispersion diagram (the multivalued function $k(\omega)$, where k is the wavenumber along the longitudinal coordinate) is of considerable importance for the description of transient properties in waveguides. Generally, the structure of the Riemann surface is not known. In the current paper we describe the structure of the Riemann surface for a non-trivial case of a three-layer waveguide. The tool for studying this Riemann surface is a gradual switching on the links between the layers. If the links are weak, the structure of the Riemann surface is found from an asymptotic consideration. While the link becomes stronger, the surface changes homotopically, in particular, the branch points travel along some continuous trajectories in the ω plane. This evolution can be tracked numerically. Thus, we bring some order into the structure of the Riemann surface of the dispersion diagram and provide a numerical technique for finding the positions of its branch points.

We should note that the case of three-layer waveguide is studied for the sake of simplicity. All results can be generalized to the case of an n-layer waveguide in a straightforward way.

The results of the paper are not yet enough to describe all transient phenomena in a waveguide analytically. One can expect that such a description can be obtained if a saddle point method is developed for a series–integral representation (see below). Currently, no such method is known. The difficulty is that the integration is held over an infinite set of contours, so the standard approach is no longer efficient. We are planning to address this problem in future. However, a study of the Riemann surface of the dispersion diagram is a necessary step for this, and we perform it here.

The structure of the paper is as follows. In Section 2 the problem for the waveguide is formulated. A dispersion equation for the waveguide is derived. The main ideas of the Miklowitz–Randles' method proposed in [13,14,12] are listed briefly. Namely, it is explained why it is important to study the analytical continuation of the dispersion diagram.

In Section 3 an example of dispersion diagram is built. It is shown that the group velocities are considerably lower than the wave velocity in the fastest layer. Thus, one can expect existence of a precursor traveling with the velocity of the fastest layer. An analytical continuation of the dispersion diagram to the complex domain of frequencies is built. It is shown that as the imaginary part of frequency grows, the avoiding-crossings of the terraced part of the diagram tend to crossing, providing the branch points of the diagram. After passing the branch points the structure of the dispersion diagram changes. The types of waveguide modes related to different branches of the diagram (real or continued) are discussed.

In Section 4 a method to study the Riemann surface of the dispersion diagram is presented. The initial problem is embedded into an analytical family of auxiliary problems depending on the linking parameters. The Riemann surface of the problem corresponding to small linking parameters can be easily constructed. A numerical method of finding the trajectories of the branch points as the linking parameters grow from small values to infinity is proposed and implemented.

In Section 5 a numerical demonstration of the Miklowitz–Randles method is presented. It is shown that fast components of the signals (the precursors) can be described by a small amount of terms in the series–integral representation after the contour is deformed.

In Appendix A a set of bilinear relations for the waveguide modes is constructed. The most important of them is the orthogonality relation for the mode corresponding to a branch point of the dispersion diagram. In Appendix B the branch points' positions are found for small linking parameters using the method of perturbations.

2. A sample three-layer waveguide and its dispersion diagram

2.1. Problem formulation

Consider a planar waveguide consisting of three liquid layers. In the (x, y)-plane the layers 1, 2, 3 occupy the strips $0 < y < H_1, H_1 < y < H_2, H_2 < y < H_3$, respectively (see Fig. 1). The densities of the media are equal to ρ_1, ρ_2, ρ_3. The sound velocities are equal to c_1, c_2, c_3. The thicknesses of the layers are

$$h_1 = H_1, \qquad h_2 = H_2 - H_1, \qquad h_3 = H_3 - H_2.$$

It is convenient to define the functions

$$c(y) = \begin{cases} c_1, & 0 < y < H_1 \\ c_2, & H_1 < y < H_2 \\ c_3, & H_2 < y < H_3 \end{cases} \qquad \rho(y) = \begin{cases} \rho_1, & 0 < y < H_1 \\ \rho_2, & H_1 < y < H_2 \\ \rho_3, & H_2 < y < H_3 \end{cases}$$