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Rayleigh waves in anisotropic porous media and the polarization vector method

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HIGHLIGHTS

- Rayleigh waves in orthotropic non-viscous fluid-saturated porous half-spaces are investigated.
- The surface of half-spaces is impervious or with sealed surface-pores.
- The secular equations in explicit form have been derived using the polarization vector method.
- The wave velocity depends strongly on the material parameters, the anisotropy and the boundary conditions.
- It may be larger, even much larger than the shear wave velocity.

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ABSTRACT

In this paper, the propagation of Rayleigh waves in orthotropic non-viscous fluid-saturated porous half-spaces with sealed surface-pores and with impervious surface is investigated. The main aim of the investigation is to derive explicit secular equations and based on them to examine the effect of the material parameters and the boundary conditions on the propagation of Rayleigh waves. By employing the method of polarization vector the explicit secular equations have been derived. These equations recover the ones corresponding to Rayleigh waves propagating in purely elastic half-spaces. It is shown from numerical examples that the Rayleigh wave velocity depends strongly on the porosity, the elastic constants, the anisotropy, the boundary conditions and it differs considerably from the one corresponding to purely elastic half-spaces. Remarkably, in the fluid saturated porous half-spaces, Rayleigh waves may travel with a larger velocity than that of the shear wave, a fact that is impossible for the purely elastic half-spaces.

1. Introduction

The investigation of surface Rayleigh wave propagation in fluid-saturated porous half-spaces is significant in many fields such as soil dynamics, earthquake engineering, geophysics and hydrology because the geological material can be considered as a type of fluid-saturated porous material (FSPM). Recently, bones are also modeled as FSPM [1]. To solve problems of FSPM one can used either the poroelasticity equations derived by Biot [2–4] or the equations obtained by using the homogenization method [5–8]. Due to the latter, the porous material can be considered as a homogenized material. As demonstrated in works [5–9], the equations derived by the homogenization method coincide with Biot's equations when the dimensionless viscosity of the fluid is small. In this paper, we will use the Biot model as the fluid is assumed to be non-viscus and hence, the two models are equivalent.

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For any investigation of Rayleigh waves, finding the explicit secular equation that determines the Rayleigh wave velocity is always the first and main purpose due to two reasons. First, the wave velocity is present in expressions of displacement and stress of Rayleigh waves. Hence, these quantities can be determined only when the Rayleigh wave velocity is known. Second, the explicit secular equations of Rayleigh waves are used as mathematical base to identify the material parameters from the experimentally measured values of Rayleigh wave velocities.

The first explicit secular equation of Rayleigh waves propagating in fluid saturated porous half spaces was derived by Johns [10] for the isotropic case with pervious boundary condition. However, one flaw in this secular equation is that only one kind of longitudinal waves was accounted for in the expressions of displacement and stress. As we know, there are two kind of longitudinal waves, fast and slow, for fluid saturated porous media. Tajuddin [11] obtained the correct explicit secular equations of Rayleigh waves in isotropic fluid saturated porous half-spaces by considering both longitudinal waves. However, these secular equations are still in determinant form, and therefore are not convenient in use. In Tajuddin's investigation, the surface of half-spaces was assumed to be pervious and impervious. The explicit secular equations in compact form of Rayleigh waves propagating in isotropic fluid saturated porous half-spaces were derived by Yang [12] for the pervious (fully opened) boundary condition and by Sharma [13] for pervious and fully closed surfaces.

The propagation of Rayleigh waves in anisotropic fluid saturated porous half-spaces was considered by Liu and Liu [14], Liu et al. [15] for the transversely isotropic case, and by Liu and Liu [16] for the orthotropic case. The authors derived the secular equations. However, all of them are in implicit form.

For the isotropic fluid saturated porous media, the characteristic equation (of six order) of Rayleigh waves can be solved analytically, three (characteristic) solutions (among six) with positive imaginary parts can therefore be defined. Consequently, the displacement and stress fields of Rayleigh waves that satisfy the decay condition can be derived explicitly. Introducing their explicit expressions into the boundary conditions, one can obtain explicit secular equations of Rayleigh waves, as demonstrated by Tajuddin [11], Yang [12], Sharma [13].

When the fluid saturated porous media are anisotropic, the situation is quite different: it is impossible to define a characteristic solution which has positive imaginary part. Therefore, for the anisotropic case, the explicit secular equations of Rayleigh waves cannot be derived by the conventional method as done by Tajuddin [11], Yang [12] and Sharma [13].

Recently, by using the method of polarization vector [17,18], one that is not based on the characteristic equation, Vinh et al. [19] obtained the explicit secular equation of Rayleigh waves propagating in orthotropic non-viscous fluid-saturated porous half-spaces *whose surface are pervious*.

In practical problems, the surface of fluid-saturated porous half-spaces may be pervious (fully opened) [11–13] or impervious [11,20] or fully closed [13,20]. Therefore, it is necessary to get explicit secular equations of Rayleigh waves for two remaining cases. To derive the explicit secular equations we also have to use the method of polarization vector because the conventional method is not applicable, as mentioned above. The method of polarization vector is based on the Stroh formalism [21] and the generalized polarization vector (the displacement-traction vector at the surface of half-spaces). For the impervious surface case, the Stroh formalism is the same as the one for the pervious case (see Vinh et al. [19]), but the generalized polarization vector is quite different from that of the pervious case. This difference makes the deriving of explicit secular equation more complicated and more difficult. For the sealed surface-pores case, we have to establish a new Stroh formalism so that the last three components of the generalized polarization vector vanish.

The obtained secular equations are purely real equations that recover the corresponding dispersion equations of Rayleigh waves propagating in pure elastic half-spaces. Since obtained secular equations are totally explicit, they are very useful in various practical applications.

Based on the obtained explicit secular equations, some numerical examples are carried out to demonstrate that the Rayleigh wave velocity depends strongly on the porosity, the material parameters and the anisotropy and that it differs considerably from the one corresponding to purely elastic half-spaces. It is also shown that the boundary conditions on the surface of half-spaces have a significant impact on the Rayleigh wave propagation. Remarkably, it is shown that, *for the elastic fluid saturated porous half-spaces, the Rayleigh wave velocity may be larger than the shear wave velocity, a fact impossible for the purely elastic half-spaces.*

2. Basic equations and boundary conditions

Following Biot [3], the equations of motion for anisotropic non-viscous-fluid-saturated porous media can be written in terms of Cartesian co-ordinates x_k as (see also [16]):

$$\sigma_{ij,j} = \rho \ddot{u}_i + \rho_f \ddot{w}_i, \;\; -p_{,i} = \rho_f \ddot{u}_i + \frac{\rho_f}{\phi} \ddot{w}_i, \;\; i = 1, 2, 3 \tag{1}$$

where a dot signifies differentiation with respect to t, a comma indicates differentiation with respect to spatial variables x_k, p is the pore pressure in the fluid assumed to be non-viscous, σ_{ij} are the total stress components, U_i and u_i are the displacement components of the saturated pore fluid and the solid skeleton, respectively, ϕ is the reference porosity, $w_i = \phi(U_i - u_i)$ is the displacement components of the fluid relative to the solid skeleton, ρ_s and ρ_f are the densities of the solid skeleton and the pore fluid and $\rho = (1 - \phi)\rho_s + \phi\rho_f$ is the composite density. The constitutive equations for anisotropic non-viscous-fluid-saturated porous media are of the form [16]:

$$\sigma_{ij} = A_{ijkl} e_{kl} + M_{ij} \zeta, \;\; p = M_{ij} e_{ij} + M\zeta \tag{2}$$