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# Proton's electromagnetic form factors from a non-power confinement potential

M. Kirchbach [a], C.B. Compean [b]

[a] Instituto de Física, Universidad Autońoma de San Luis Potosí, Av. Manuel Nava 6, Zona Universitaria, San Luis Potosí, S.L.P. 78290, Mexico
[b] Facultad de Ciencias, Universidad Autónoma de San Luis Potosí, Lateral Av. Salvador Nava s/n, San Luis Potosí, S.L.P. 78290, Mexico

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## Abstract

The electric-charge, and magnetic-dipole form factors of the proton are calculated from an underlying constituent quark picture of hadron structure based on a potential shaped after a cotangent function, which has the properties of being both conformally symmetric and color confining, finding adequate reproduction of a variety of related data.


*Keywords:* General properties of QCD; Potential models; Electric and magnetic moments

## 1. Introduction

Constituent quark model descriptions of hadron properties, such as excitation spectra, decay modes, or electromagnetic form-factors, employ quantum few-body problems techniques based on effective potentials [1] supposed to capture to some extent the essentials of the fundamental confining strong interaction. The potentials of widest spread in the literature are shaped after power-functions of the relative distances between the quarks, and among them one encounters for example (i) the infinite power square well potential, $V_{SW}(r) = 1/r^{\infty} = 0, \quad 0 \le r \le r_0$, and

*E-mail address:* mariana@ifisica.uaslp.mx (M. Kirchbach).

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$V(r) = r^\infty = \infty$ for $\infty < r < 0$ and $r > r_0$, (ii) the harmonic oscillator, $V_{HO}(r) = \omega^2 r^2/2$, (iii) the Cornell potential, $V_C(r) = -\alpha/r + \beta r$, etc. Such models usually require a significant number of free parameters to produce wave functions of the quark systems capable to account for the specific of a variety of data compiled in [2]. One of the reason for this circumstance can be related to the mismatch between the symmetry properties of the power-potentials and the symmetries of the fundamental strong interaction like the conformal symmetry, which manifests itself among others by the walking of the strong coupling $\alpha_s$ to a fixed value in the infrared regime of QCD [3] and the notable hydrogen-like degeneracies appearing in the mass distributions of the unflavored mesons, a phenomenon addressed for example in [4], [5], [6]. In order to improve this aspect of the quark models, it naturally comes to ones mind to explore more complicated potential functions. The observation that the infinite square well has same spectrum as the $\left[2\csc^2(\pi r/r_0) - 1\right]$ potential, referred to in [7] as the "super-symmetric partner" to $V_{SW}(r/r_0)$, seems to point toward the exactly solvable trigonometric potentials known from the super-symmetric quantum mechanics (SUSY-QM), as possible upgrades to the power-potentials. Indeed, several of the finite power-potential series in use can be viewed as first terms in the infinite series expansions of properly designed trigonometric functions. Specifically, the (here dimensionless) inverse square distance term, $R^2/r^2$ where $R$ is a matching length parameter, approximates $\csc^2(r/R)$ at large $(r/R)$ values. The linear plus harmonic oscillator potential can be viewed as an approximation to the trigonometric Scarf potential,

$$V_{tSc}\left(\frac{r}{R}\right) = [b^2 + a(a+1)]\sec^2\left(\frac{r}{R}\right) - b(2a+1)\sec\left(\frac{r}{R}\right)\tan\left(\frac{r}{R}\right)$$
$$\approx -b(2a+1)\frac{r}{R} + [b^2 + a(a+1)]\left(\frac{r}{R}\right)^2, \tag{1}$$

by the first terms of its series expansion, while the Cornell potential could be viewed as a truncation of the series expansion of the cotangent function according to,

$$-b\cot\left(\frac{r}{R}\right) \approx -b\frac{R}{r} + \frac{b}{3}\frac{r}{R}. \tag{2}$$

The principal advantage of trigonometric- over finite power potentials lies not that much in the exact solubility of the former, but rather in their symmetry properties, which show up in certain appropriately chosen variables. For example, while the centrifugal barrier, $\ell(\ell+1)/r^2$, and the Cornell potential are only rotationally symmetric, their trigonometric extensions towards $\ell(\ell+1)$ $\csc^2(r/R)$ and $-\cot(r/R)$ have the higher $O(4)$ symmetry, just as would be required by the conformal symmetry at the level of the excitations. This is visible from the fact that the stationary Schrödinger wave equation, describing (upon separation of center-of mass and relative, $r/R$, coordinates) the one-dimensional radial part

$$\left[-\frac{\hbar^2 c^2}{R^2}\frac{d^2}{d\chi^2} + \frac{\hbar^2 c^2}{R^2}\frac{\ell(\ell+1)}{\sin^2\chi} - 2\frac{\hbar^2 c^2 b^2}{R^2}\cot\chi\right]U_{n\ell}(\chi) = \mathcal{E}^2 U_{n\ell}(\chi),\ \ \chi = \frac{r}{R} \in [0,\pi], \tag{3}$$

of a two-body wave function, with $n$ being the node-number, and $\ell$ the relative angular momentum value, can be transformed through the change

$$U_{n\ell}(\chi)Y_\ell^m(\theta,\varphi) = \frac{\Psi_{n\ell}(\chi)}{\sin\chi}Y_\ell^m(\theta,\varphi) = \Psi_{n\ell}^{tot}(\chi,\theta,\varphi), \tag{4}$$

to quantum motion on the three dimensional hypersphere, $S^3$, according to

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