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Method for designing error-resistant phase-shifting algorithm

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ABSTRACT

We present a method for designing error-resistant phase-shifting algorithms to suppress error sources in phase-shifting interferometry. Firstly, the partial-differential processing is applied to the weighted least squares algorithm to obtain error sensitivity equations. Sequentially, bound equations are obtained to minimize error sensitivity. Finally, the bound equations are solved to determine the weights, and the error-resistant phase-shifting algorithms are developed. Aiming at a self-developed interferometer, the proposed method is used to design phase-shifting algorithms which are resistant to given error sources. Theoretical analysis and numerical simulations of the self-designed algorithms compared with a commercial algorithm are completed. Theoretical analysis indicates that the self-designed algorithms meet the desired requirements. Numerical simulations verify the correctness of theoretical analysis. And the comparisons show that the self-designed algorithm is more resistant to error sources that needs to be suppressed. These results verify the proposed method and demonstrate its effectiveness.

1. Introduction

In phase-shifting interferometry, the measured object is the phase difference of reference and test wave fields. Phase-shifter introduces additional OPD (optical path difference), which yields sequential interferograms. These interferograms are captured by detector, and then are processed using a given phase-shifting algorithm to calculate the phase difference [1]. This technique reduces the influence of contrast, and yields good results even if the interferograms have poor contrast. Furthermore, it reduces the influence of background and nonuniformity of the light source, and provides high accuracy [2].

In phase-shifting interferometers, PZT (piezoelectric transducers) is commonly used as phase shifter, CCD (charge-coupled device) or CMOS (complementary metal oxide semiconductor) is used as detector, and laser is typically used as light source. The nonideal performance of these devices usually causes measurement errors. Therefore, the phase-shifting algorithm should be designed to suppress these error sources.

To date, many methods have been reported on designing phase-shifting algorithms to minimize particular errors. Freischlad [3] proposed a method to evaluate the performance of phase-shifting algorithms through Fourier theory and Zhang [4] used this theory to derive a new error-resistant algorithm. De Groot [5] designed algorithms via window functions such as the Hanning window. Surrel [6] presented a characteristic polynomial theory for designing phase-shifting algorithms, and Zhu [7] designed a new algorithm by overlapping averaged results of the old algorithm, making the new algorithm more insensitive to phase-shift error. Phillion [8] proposed a method based on recursion rules to design a new algorithm from the old one. Shi [9] presented an effective approach to derive phase-shifting algorithms based on the self-convolution of a rectangle window, and designed an algorithm to suppress the phase-shift error and detector-response error simultaneously. However, to the best of our knowledge, these methods do not discuss the error sensitivity of algorithms, but only focus on suppressing phase-shift error, detector-response error, or both, while ignoring the other error sources.

This paper thus proposes an effective method for designing error-resistant algorithms. By applying the partial-differential method, we get sensitivity relationship between weighted least-square algorithm and error sources, which named error sensitivity equations. When the error sensitivity vanishes, the weighted least-square algorithm is insensitive to corresponding error sources. And then, we get bound equations. Sequentially, the bound equations are solved to determine the weights, and finally, phase-shifting algorithms are obtained that are resistant to

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corresponding error sources. The designed error-resistant phase-shifting algorithms effectively reduce the performance requirements for phase shifter, detector, and light source.

This paper is organized as follows. Section 2 introduces the weighted least-square phase-shifting algorithm. Section 3 introduces the main error sources (phase-shift error, detector-response error, and light-source-instability), analyses sensitivity relationship between weighted least-square algorithm and these error sources, and then derives the bound equations. Section 4 takes a self-designed interferometer as an example to design algorithms which are capable of suppressing the first-, second-, and third-order phase-shift error, the first- and second-order detector-response error, and the first- and second-order light-intensity–instability. Section 5 uses Fourier transform theory and numerical simulations to evaluate the performance of self-designed algorithms. The self-designed algorithms are compared with Zygo 13-frames algorithm [10,11], and the results demonstrate the effectiveness of the proposed method.

2. Weighted least-square algorithm

In phase-shifting interferometry, the nth irradiance $I_n(x,y)$ at a point (x,y) could be expressed as follows [12],

$$I_n(x,y) = A(x,y) + B(x,y)\times\cos\left[\phi(x,y) + \delta_n(x,y)\right] \tag{1}$$

where $A(x,y)$ is the background intensity, $B(x,y)$ is the amplitude of modulation, the quantity to be measured $\phi(x,y)$ is the phase difference of the wave fields that interfere, and $\delta_n(x,y)$ is the phase shift of the nth irradiance. Eq. (1) can be rewritten as,

$$I_n(x,y) = A(x,y) + B_{cos}(x,y)\cos\left[\delta_n(x,y)\right] + B_{sin}(x,y)\sin\left[\delta_n(x,y)\right] \tag{2}$$

where,

$$\begin{aligned} B_{cos}(x,y) &= B(x,y)\cos\left[\phi(x,y)\right] \\ B_{sin}(x,y) &= -B(x,y)\sin\left[\phi(x,y)\right] \end{aligned} \tag{3}$$

Then, the phase difference can be calculated from $B_{sin}(x,y)$ and $B_{cos}(x,y)$ by,

$$\phi(x,y) = \arctan\left[-\frac{B_{sin}(x,y)}{B_{cos}(x,y)}\right] \tag{4}$$

The phase at point (x,y) is only determined by the intensity and phase shift at this point, so we omit the explicit dependence (x,y) on position. Phase-shifting algorithm can be developed from the principle of weighted least-square estimation [13]. With the weight function, the error function ϵ could be defined as,

$$\epsilon = \sum_{n=1}^{N} w_n\left(I_n - \widetilde{I}_n\right)^2 = \sum_{n=1}^{N} w_n\left[A + B_{cos}\cos(\delta_n) + B_{sin}\sin(\delta_n) - \widetilde{I}_n\right]^2 \tag{5}$$

where $\widetilde{I}_n$ represents the nth actual irradiance and w_n is the nth weight. The ϵ is minimized when the derivatives of ϵ with respect to A, B_{sin} and B_{cos} vanish. This condition yields the following matrix equation,

$$\begin{bmatrix} \sum_{n=1}^{N} w_n & \sum_{n=1}^{N} w_n\cos(\delta_n) & \sum_{n=1}^{N} w_n\sin(\delta_n) \\ \sum_{n=1}^{N} w_n\cos(\delta_n) & \sum_{n=1}^{N} w_n\cos^2(\delta_n) & \sum_{n=1}^{N} w_n\sin(\delta_n)\cos(\delta_n) \\ \sum_{n=1}^{N} w_n\sin(\delta_n) & \sum_{n=1}^{N} w_n\sin(\delta_n)\cos(\delta_n) & \sum_{n=1}^{N} w_n\sin^2(\delta_n) \end{bmatrix} \times \begin{bmatrix} A \\ B_{cos} \\ B_{sin} \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^{N} w_n\widetilde{I}_n \\ \sum_{n=1}^{N} w_n\widetilde{I}_n\cos(\delta_n) \\ \sum_{n=1}^{N} w_n\widetilde{I}_n\sin(\delta_n) \end{bmatrix} \tag{6}$$

If the weights are selected to satisfy the following conditions,

$$\begin{cases} \sum_{n=1}^{N} w_n = 1 \\ \sum_{n=1}^{N} w_n\cos(\delta_n) = \sum_{n=1}^{N} w_n\sin(\delta_n) = \sum_{n=1}^{N} w_n\sin\left(2\delta_n\right) = 0 \\ \sum_{n=1}^{N} w_n\cos^2(\delta_n) = \sum_{n=1}^{N} w_n\sin^2(\delta_n) = Q \end{cases} \tag{7}$$

where Q is a non-zero constant, the phase difference ϕ can be calculated by,

$$\phi = \arctan\left[-\frac{\sum_{n=1}^{N} w_n\widetilde{I}_n\sin(\delta_n)}{\sum_{n=1}^{N} w_n\widetilde{I}_n\cos(\delta_n)}\right] \tag{8}$$

Eq. (8) is the formula of weighted phase-shifting algorithm. When appropriate weights are selected, weighted phase-shifting algorithm could be resistant to given error sources.

3. Sensitivity analysis for error sources

In this section, we analyze sensitivity of weighted phase-shifting algorithm to phase-shift error, detector-response error, and light-source-instability, which are main error sources in phase-shift interferometry. And finally, we obtain the bound equations to minimize the influence of these error sources.

3.1. Phase-shift error

In the case of a linear or nonlinear miscalibration, the actual nth phase shift δ_n' may be expressed as a polynomial of ideal phase shift δ_n [14]. Note that in the first irradiance, phase shift is zero, so $\delta_1' = \delta_1 = 0$. For equal-interval phase shift, $\delta_n = (n-1)\delta$, where δ is single phase shift interval. When k order phase-shift errors exist, δ_n' is a k order polynomial of frames number n, as,

$$\delta_n' = \left[\left(1+\zeta_1\right)(n-1) + \zeta_2(n-1)^2 + \cdots + \zeta_k(n-1)^k\right]\delta \tag{9}$$

where, ζ_k is the coefficient of the kth order phase-shift error. The phase-shift error defined as the difference between δ_n' and δ_n is,

$$\Delta\delta_n = \left[\zeta_1(n-1) + \zeta_2(n-1)^2 + \cdots + \zeta_k(n-1)^k\right]\delta \tag{10}$$

From the partial derivative of Eq. (8) with respect to δ_n, we get phase extraction error caused by phase-shift error (see Box I). Using Eq. (7), the denominator and numerator of Eq. (11) can be simplified respectively as,

$$\begin{aligned} denominator = &\left[B\cos(\phi)\sum_{n=1}^{N} w_n I_n\cos^2\left(\delta_n\right)\right]^2 \\ &+ \left[-B\sin(\phi)\sum_{n=1}^{N} w_n I_n\sin^2\left(\delta_n\right)\right]^2 \\ &= B^2Q^2 \end{aligned} \tag{12}$$

and,

$$\begin{aligned} numerator = B^2Q\Bigg\{&\cos(\phi)\left[\frac{\sin(\phi)}{2}\sum_{n=1}^{N} w_n\sin\left(2\delta_n\right)\Delta\delta_n + \cos(\phi)\sum_{n=1}^{N} w_n\sin^2\left(\delta_n\right)\Delta\delta_n\right] \\ &\sin(\phi)\left[\frac{\cos(\phi)}{2}\sum_{n=1}^{N} w_n\sin\left(2\delta_n\right)\Delta\delta_n + \sin(\phi)\sum_{n=1}^{N} w_n\cos^2\left(\delta_n\right)\Delta\delta_n\right]\Bigg\} \end{aligned} \tag{13}$$